

1. Without simplifying, state the terms in the expansion of each of the following.

- (a) $(a + b)^5$
- (b) $(x + y)^3$
- (c) $\left(\frac{1}{4} + \frac{3}{4}\right)^4$

2. If the numerical coefficients are disregarded, terms of the following form will appear in the expansion of $(a + b)^9$. State the value of the exponent k in each case.

- (a) a^3b^k
- (b) a^kb^8
- (c) a^9b^k
- (d) $a^{k+1}b^k$

3. State the numerical coefficient, in the form $\binom{n}{r}$, for each of the following terms in the expansion of $(a + b)^{11}$.

- (a) a^9b^2
- (b) a^3b^8
- (c) a^{11}
- (d) $a^{11-r}b^r$

4. Express each of the following in the form $(a + b)^n$.

- (a) $\binom{4}{0}a^4 + \binom{4}{1}a^3b + \binom{4}{2}a^2b^2 + \binom{4}{3}ab^3 + \binom{4}{4}b^4$
- (b) $\binom{6}{0}(x^2)^6 + \binom{6}{1}(x^2)^5\left(\frac{1}{x}\right) + \binom{6}{2}(x^2)^4\left(\frac{1}{x}\right)^2 + \dots + \binom{6}{6}\left(\frac{1}{x}\right)^6$
- (c) $\binom{5}{0}(-3)^5 + \binom{5}{1}(-3)^4(2) + \binom{5}{2}(-3)^3(2)^2 + \binom{5}{3}(-3)^2(2)^3 + \binom{5}{4}(-3)(2)^4 + \binom{5}{5}(2)^5$

5. Expand and simplify each of the following.

- (a) $(2a + b)^3$
- (b) $(a - 2b)^4$
- (c) $(1 - x)^5$
- (d) $(1 + x^2)^6$
- (e) $\left(1 + \frac{1}{x}\right)^4$
- (f) $\left(x - \frac{1}{x}\right)^5$

6. Expand and simplify each of the following.

- (a) $\left(x - \frac{2}{x^2}\right)^5$
- (b) $(2x^3 + \sqrt{y})^4$
- (c) $\left(a^2 + \frac{3b}{a}\right)^4$
- (d) $\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)^6$

7. Find the first four terms in the expansion of each of the following.

- (a) $(a + b)^{10}$
- (b) $(1 - x^2)^{12}$
- (c) $\left(x^2 + \frac{2}{x^2}\right)^9$
- (d) $\left(2x - \frac{3}{x^2}\right)^8$
- (e) $\left(x^3 - \frac{2}{x^2}\right)^6$
- (f) $\left(x + \sqrt{x^3}\right)^{11}$

8. Evaluate each of the following.

$$(a) \binom{5}{0}\left(\frac{1}{3}\right)^5 + \binom{5}{1}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right) + \binom{5}{2}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2 + \binom{5}{3}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^3 \\ + \binom{5}{4}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^4 + \binom{5}{5}\left(\frac{2}{3}\right)^5$$

$$(b) \binom{4}{0}2^4 + \binom{4}{1}2^3(-1) + \binom{4}{2}2^2(-1)^2 + \binom{4}{3}2(-1)^3 \\ + \binom{4}{4}(-1)^4$$

$$(c) \binom{3}{0}1^3 + \binom{3}{1}1^2(-3) + \binom{3}{2}1(-3)^2 + \binom{3}{3}(-3)^3$$

9. Write the Binomial Theorem by using sigma notation.

14. In the expansion of $(1 + x)^n$, the first three terms are $1 - 18 + 144$. Find the values of x and n .

15. In the expansion of $(1 + ax)^n$, the first three terms are $1 + \frac{5}{3}x + \frac{10}{9}x^2$. Find the value of a and n .

16. The polynomial $(p + q)^9$ is expanded in decreasing powers of p . The second and third terms have equal values, where p and q are positive numbers whose sum is one. What is the value of p ?

Answers

$$1. (a) \binom{5}{0}a^5 + \binom{5}{1}a^4b + \binom{5}{2}a^3b^2 + \binom{5}{3}a^2b^3 \\ + \binom{5}{4}ab^4 + \binom{5}{5}b^5 \quad (b) \binom{3}{0}x^3 + \binom{3}{1}x^2y + \binom{3}{2}xy^2 + \binom{3}{3}y^3 \quad (c) \binom{4}{0}\left(\frac{1}{4}\right)^4 + \binom{4}{1}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right) \\ + \binom{4}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^2 + \binom{4}{3}\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^3 + \binom{4}{4}\left(\frac{3}{4}\right)^4$$

$$2. (a) 6 \quad (b) 1 \quad (c) 0 \quad (d) 4$$

$$3. (a) \binom{11}{2} \quad (b) \binom{11}{8} \quad (c) \binom{11}{0} \quad (d) \binom{11}{r}$$

$$4. (a) (a + b)^4 \quad (b) (x^2 + \frac{1}{x})^6 \quad (c) (-3 + 2)^5$$

$$5. (a) 8a^3 + 12a^2b + 6ab^2 + b^3 \\ (b) a^4 - 8a^3b + 24a^2b^2 - 32ab^3 + 16b^4 \\ (c) 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5 \\ (d) 1 + 6x^2 + 15x^4 + 20x^6 + 15x^8 + 6x^{10} + x^{12}$$

$$(e) 1 + \frac{4}{x} + \frac{6}{x^2} + \frac{4}{x^3} + \frac{1}{x^4}$$

$$(f) x^5 - 5x^3 + 10x - \frac{10}{x} + \frac{5}{x^3} - \frac{1}{x^5}$$

$$6. (a) x^5 - 10x^2 + \frac{40}{x} - \frac{80}{x^4} + \frac{80}{x^7} - \frac{32}{x^{10}}$$

$$(b) 16x^{12} + 32x^9\sqrt{y} + 24x^6y + 8x^3\sqrt{y^3} + y^2$$

$$(c) a^8 + 12a^5b + 54a^2b^2 + \frac{108b^3}{a} + \frac{81b^4}{a^4}$$

$$(d) x^3 - 12x^2 + 60x - 160 + \frac{240}{x} - \frac{192}{x^2} + \frac{64}{x^3}$$

$$7. (a) a^{10} + 10a^9b + 45a^8b^2 + 120a^7b^3 + \dots$$

$$(b) 1 - 12x^2 + 66x^4 - 220x^6 + \dots$$

$$(c) x^{18} + 18x^{14} + 144x^{10} + 672x^6 + \dots$$

$$(d) 256x^8 - 3072x^5 + 16128x^2 - \frac{48384}{x} + \dots$$

$$(e) x^{18} - 12x^{13} + 60x^8 - 160x^3 + \dots$$

$$(f) x^{11} + 11\sqrt{x^{23}} + 55x^{12} + 165\sqrt{x^{25}} + \dots$$

$$8. (a) 1 \quad (b) 1 \quad (c) -8$$

$$9. (a + b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$$

$$14. n=9, x=-2 \quad 15. n=5, a=\frac{1}{3} \quad 16. \frac{4}{5}$$