Arithmetic & Geometric Series (Answers)

Date:

Consecutive Numbers

Recall the tale of our hero, Carl Friedrich Gauss at the age of eight… (Page 466)

$$S_{100} = 1 + 2 + 3 + \ldots + 98 + 99 + 100$$

$$S_{100} = 100 + 99 + 98 + \ldots + 3 + 2 + 1$$

$$2S_{100} = 101 + 101 + 101 + \ldots + 101 + 101 + 101$$

$$= 101(100)$$

Therefore, $S_{100} = 5050$

Arithmetic Series

Use the same method to generalize and derive a formula for the sum of an arithmetic series: (Page 466)

$$S_n = a + (a + d) + (a + 2d) + \ldots + [a + (n-2)d] + [a + (n-1)d]$$

$$S_n = [a + (n-1)d] + [a + (n-2)d] + \ldots + (a + d) + a$$

$$2S_n = [2a + (n-1)d] \times n \text{ terms}$$

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d] \text{ or } S_n = \frac{n(t_1 + t_n)}{2}$$

Geometric Series

Use a similar method to generalize and derive a formula for the sum of a geometric series: (Page 473)

$$S_n = a + ar + ar^2 + \ldots + ar^{n-1}$$

$$rS_n = ar + ar^2 + \ldots + ar^{n-1} + ar^n$$

$$rS_n - S_n = ar^n - a$$

$$S_n(r-1) = a(r^n - 1)$$

$$\therefore S_n = \frac{a(r^n - 1)}{r - 1}$$