The Present Value of an Ordinary Annuity

Date:

An ***annuity*** is a sequence of equal payments made at equally spaced intervals of time.

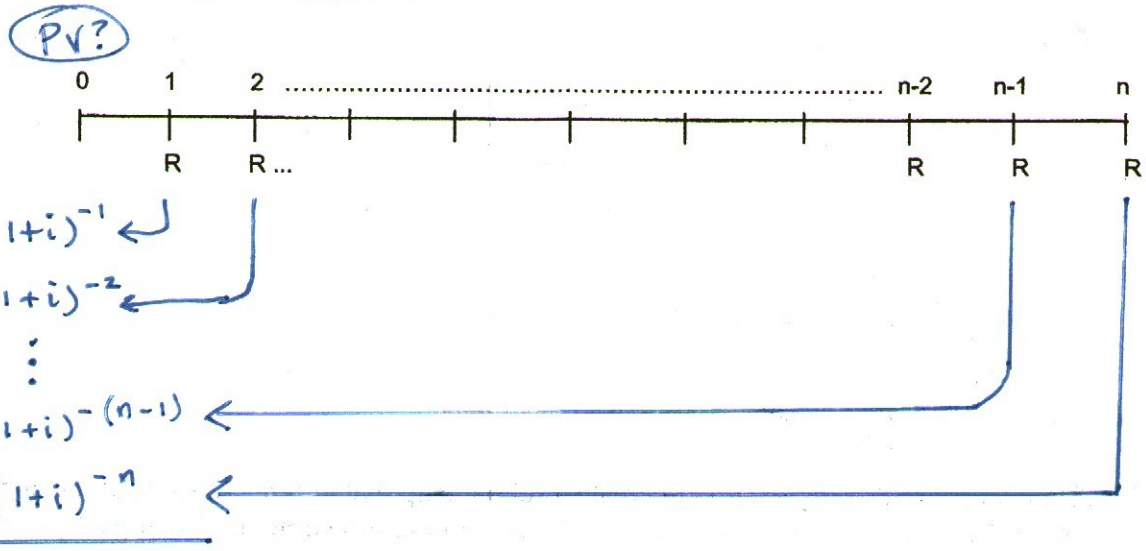
The ***period of an annuity*** is the time interval between two consecutive payments.

The ***term of an annuity*** is the total time involved in completing the annuity.

Ordinary annuities have payments made at the end of the payment period.

PV: the present value of the annuity
R: Regular payment
i: the adjusted interest rate (interest per annum ÷ the number of compounding periods in a year)
n: Number of payments in total

We will use a ***timeline*** to visualize the money in an annuity and derive a formula for the present value of an ordinary annuity after *n* compounding periods.



geo. series

$S_n = \frac{a(r^n-1)}{(r-1)}$

$a = R(1+i)^{-1}$

$r = (1+i)^{-1}$

n terms

$$PV = R(1+i)^{-1}\left[\frac{(1+i)^{-n}-1}{(1+i)^{-1}-1}\right]$$

$$= \frac{R[(1+i)^{-n}-1]}{(1+i)^{1}[(1+i)^{-1}-1]}$$

$$= \frac{R[(1+i)^{-n}-1]}{(1+i)^{0}-(1+i)}$$

$$= \frac{R[(1+i)^{-n}-1]}{1-1-i}$$

$$= \frac{R[(1+i)^{-n}-1]}{-i}$$

put the negative in numerator

$$\boxed{PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]}$$

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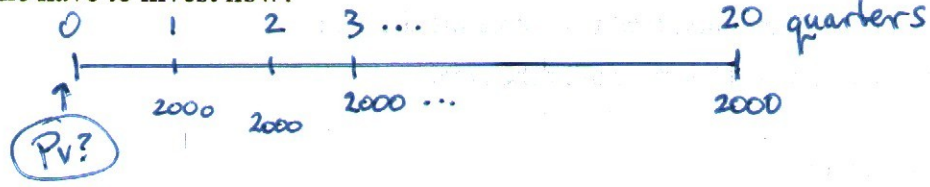
Ex. Michael wants to invest so that he can withdraw $2000 at the end of each quarter for 5 years. The first payment is due at the end of the next quarter and he can earn 3% compounded quarterly. How much does he have to invest now?



$R = 2000$

$i = \frac{0.03}{4}$ quarterly

$n = 5(4) = 20$ quarters

$$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$= 2000\left[\frac{1-\left(1+\frac{0.03}{4}\right)^{-20}}{\left(\frac{0.03}{4}\right)}\right]$$

$$= \$37\,016.04$$

Commentary:

- compare

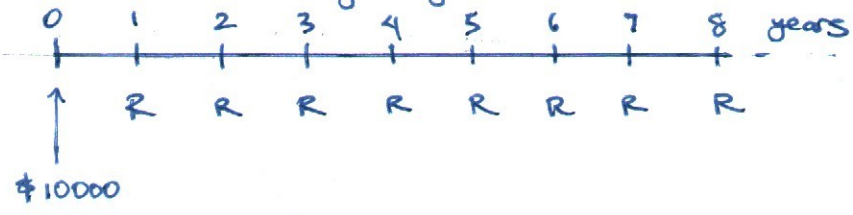
$20(\$2000) = \$40\,000$

vs.

$PV = \$37\,016.04$

Why is the PV less than $40 000

Ex. Mr. Shim wants to donate $10 000 to create a scholarship that will be awarded at the end of the year for 8 years. If he invests at the ~~end~~ beginning of the school year earning 6% compounded annually, how much will the scholarship be every year?



$i = 0.06$ annually

$$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$10000 = R\left[\frac{1+(1.06^{-8})}{0.06}\right]$$

$R \doteq 1610.36$

∴ the scholarship will be $1610.36 every year.