The Amount of an Ordinary Annuity

Date:

An ***annuity*** is a sequence of equal payments made at equally spaced intervals of time.

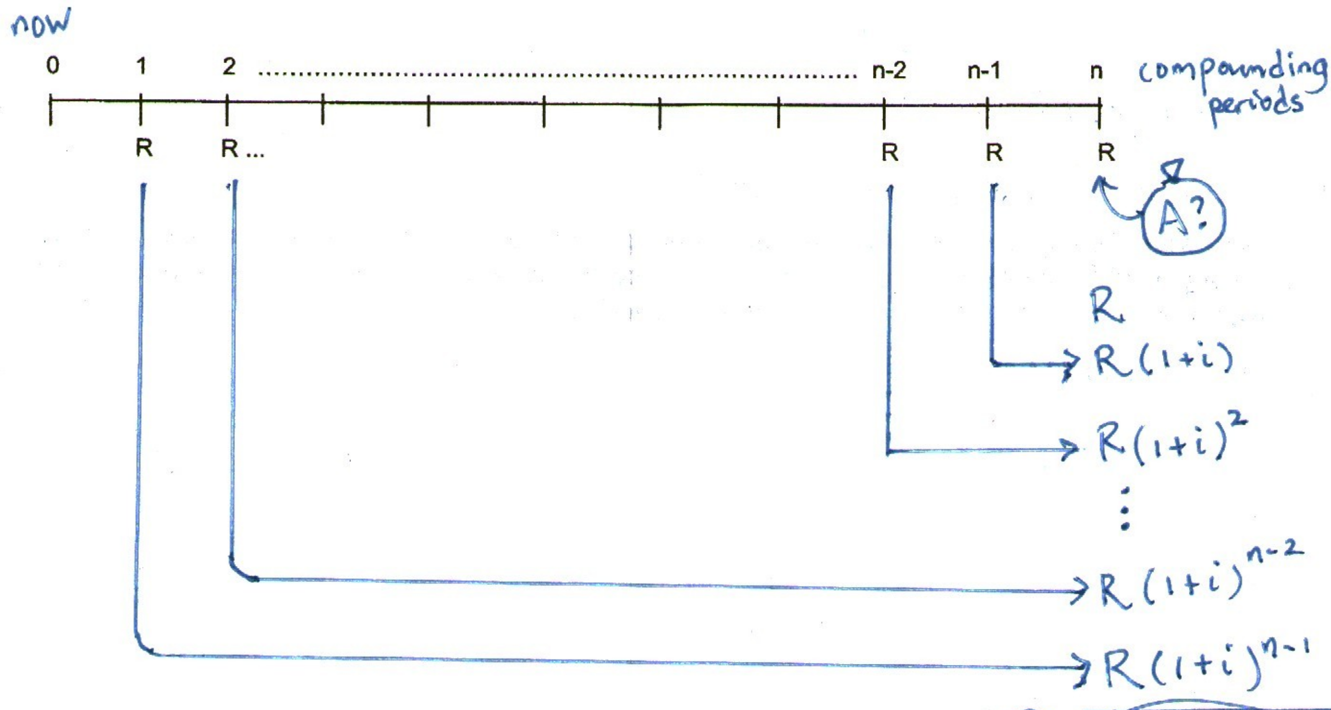
The ***period of an annuity*** is the time interval between two consecutive payments.

The ***term of an annuity*** is the total time involved in completing the annuity.

Ordinary annuities have payments made at the end of the payment period.

A: Amount of the annuity
R: Regular payment
i: the adjusted interest rate (interest per annum ÷ the number of compounding periods in a year)
n: Number of payments in total

We will use a ***timeline*** to visualize the money in an annuity and derive a formula for the future value of an ordinary annuity after n compounding periods.



A? = geo. series!

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

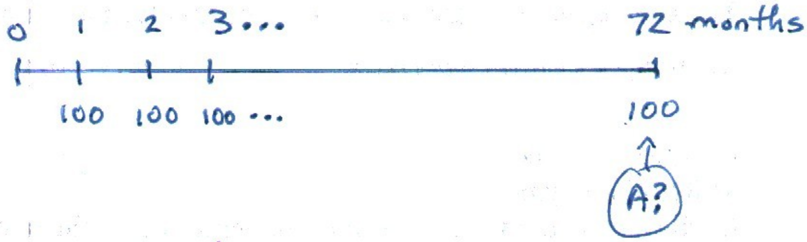
$a = R$
$r = (1+i)$
n terms

$$A = \frac{R[(1+i)^n - 1]}{(1+i) - 1}$$

$$A = \frac{R[(1+i)^n - 1]}{i}$$

Ex. 1 Finding the amount of an annuity. (A?)

If you were to save \$100 per month, earning 3% per annum, compounded monthly, then how much would you have saved at the end of 6 years?



$R = \$100$

$i = \frac{0.03}{12}$

$n = 6(12) = 72$ months

$$A = \frac{R[(1+i)^n - 1]}{i}$$

$$= \frac{100[(1+\frac{0.03}{12})^{72} - 1]}{(\frac{0.03}{12})}$$

$$\doteq \$7877.94$$

Commentary:

- compare $\$100(72) = \$7200$ vs. $A = \$7877.94$
  - ↳ difference \$677.94 is the accumulated interest

Ex. 2 Finding the monthly payment of an annuity.

Sarah wants to buy a home theatre system for her grandfather's retirement in three years. She estimates that she would spend \$4000, including taxes, where she can earn 5% per annum, compounded monthly. How much would she have to save monthly to reach this goal?

$A = \$4000$

$i = \frac{0.05}{12}$, monthly

$n = 3(12) = 36$ months

$R = ?$

$$A = \frac{R[(1+i)^n - 1]}{i}$$

$$4000 = \frac{R[(1+\frac{0.05}{12})^{36} - 1]}{(\frac{0.05}{12})}$$

$$4000 \doteq 38.75333552R$$

$$R = \$103.22$$

∴ Sarah would have to save \$103.22 per month to accumulate \$4000.