Problem Solving in Measurement **Date:**

Problem 1: If 3 lead spheres of radii 3, 4 and 5 are melted to form a new larger sphere. What is the radius of the larger sphere assuming no lead is lost in the melting process?

<u>Solution</u>

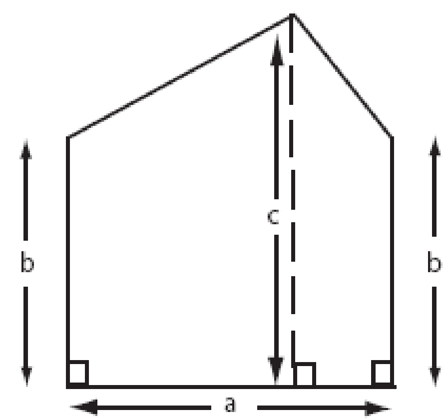
The volume of the new sphere must be sum of the volumes of the smaller spheres. If we let the radius of the new and original spheres be $R, r_1, r_2,$ *and* r_3 *respectively, then*

$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$ *or* $R^3 = r_1^3 + r_2^3 + r_3^3$, *dividing through by* $\frac{4}{3}\pi$. *Therefore*

$R^3 = 3^3 + 4^3 + 5^3 = 216$ *and R=6.*

Problem 2: What is the area of the pentagon shown?

7) The area is an a by b rectangle plus a triangle of height c-a and base b. The area is ab +a(c-b)/2= ab/2+ac/2=a(b+c)/2.



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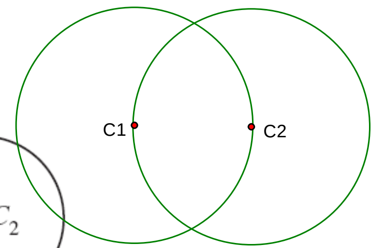
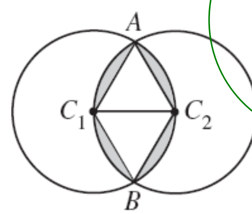
Problem 3: Two circles of radius 6 have centres C_1 and C_2 as shown. If the centre of each circle lies on the circumference of the other circle, calculate the total area covered by the two circles.



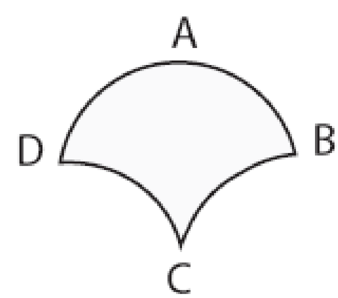
Solution

The required area is the sum of the areas of the 2 circles minus the area of overlap, which must be subtracted to account for the fact that it was counted twice, once within each circle. If we label the points of intersection of the circles A and B, then AC_1C_2 *and* BC_1C_2 *are both equilateral triangles of side 6. Each of these triangles has an altitude* $3\sqrt{3}$ *and area* $9\sqrt{3}$*. The overlap region consists of the 2 triangles plus 4 small, shaded regions between the triangles and the arcs of the circles. However a 60 degree sector of a circle of radius 6 which has 1/6 of the area the circle or* 6π*, is also 1 triangle plus 1 of these small regions. Thus the area of each small region is* $6\pi - 9\sqrt{3}$*. The region of overlap is* $18\sqrt{3} + 4(6\pi - 9\sqrt{3}) = 24\pi - 18\sqrt{3}$*. Finally the total area is* $2(36\pi) - (24\pi - 18\sqrt{3}) = 48\pi + 18\sqrt{3}$.

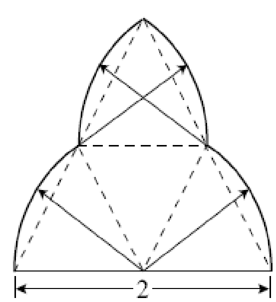


Problem 4: The figure ABCD shown in the diagram consists of 4 quarter circle arcs AB, BC, CD and DA, each of radius 4. What is the enclosed area?

Problem 4) Draw a big square around the whole thing. Area of the top two arcs is ½ a circle, so its 8pi. The area of the two bits is 2 (16 – 4pi). Area = 32.

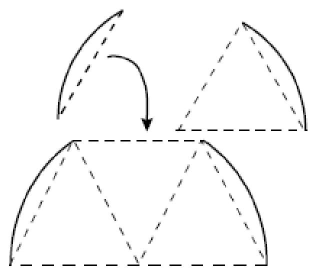


Problem 5: The figure shown is called a trefoil and is constructed by drawing circular sectors about sides of the congruent equilateral triangles. What is the area of the trefoil whose horizontal base has a length of 2?



- **Solution (B)** The trefoil is constructed of four equilateral triangles and four circular segments, as shown. These can be combined to form four $60°$ circular sectors. Since the radius of the circle is 1, the area of the trefoil is

$$\frac{4}{6}\left(\pi \cdot 1^2\right) = \frac{2}{3}\pi.$$



OR

The trefoil has the same area as four $60°$ sectors of a circle of radius 1, so it area is

$$4\left(\frac{1}{6}\right)\pi 1^2 = \frac{2}{3}\pi.$$

The figure below bounded by the solid lines shows half of the trefoil. The circular segments OBC and EDB each have area $(1/6)\pi \cdot 1^2 = \pi/6$ for a total area of $\pi/3$. The area of $\triangle EAD$ is included in this total area and should not be, and the area of $\triangle BAO$ is not included in this total area and should be. But $\triangle EAD$ is congruent to $\triangle BAO$ so half the trefoil has area $\pi/3$, and the trefoil has area $2\pi/3$.

OR

The figure below shows a partitioning of the trefoil where one of the sectors on the top has been moved above the middle equilateral triangle in the base. This shows that the area of the trefoil is $4/3$ the area of the semicircle with radius 1, that is, $(4/3)(1/2)\pi = 2\pi/3$.

Difficulty:

NCTM Standard: Problem Solving Standard for Grades 9–12: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Mathworld.com Classification:

Geometry > Plane Geometry > Arcs > Circular Sector

Problem 6: If each edge of a rectangular prism is increased by 20%, what is the percentage increase in its volume?

Since V=LWH , the new volume is (1.2)L(1.2)W(1.2)H=1.728 V.

Problem 7: A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25%, without altering the volume, by what percent must the height be decreased?

Solution (C) Let r, h, and V, respectively, be the radius, height, and volume of the jar that is currently being used. The new jar will have a radius of $1.25r$ and volume V. Let H be the height of the new jar. Then

$$\pi r^2 h = V = \pi(1.25r)^2 H, \quad \text{so} \quad \frac{H}{h} = \frac{1}{(1.25)^2} = 0.64.$$

Thus H is 64% of h, so the height must be reduced by $(100 - 64)\% = 36\%$.

OR

Multiplying the diameter by $5/4$ multiplies the area of the base by $(5/4)^2 = 25/16$, so in order to keep the same volume, the height must be multiplied by $16/25$. Thus the height must be decreased by $9/25$, or 36%.

ty: Medium-hard

Standard: Geometry Standard: Use visualization, spatial reasoning, and geometric modeling to solve problems

orld.com Classification:

ry > Solid Geometry > Cylinders > Cylinder