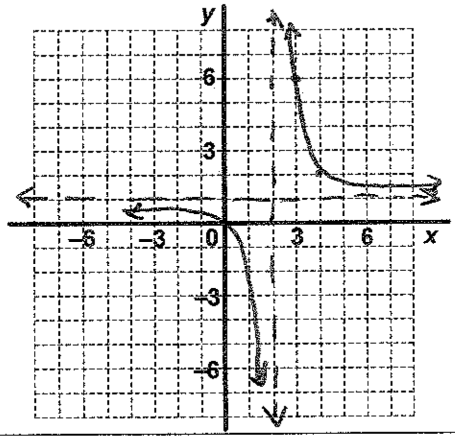
Investigating Rational Functions of the form $f(x)=\dfrac{ax+b}{cx+d}$

Date: March 10/16

Graph each of the following rational functions and fill in the blanks below (check on the TI-83 calculator).

$y=\dfrac{x+4}{x-2}$



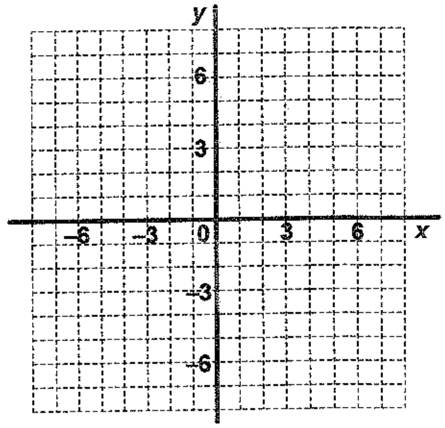
Domain : $\{x \in \mathbb{R},\ x \neq 2\}$

Vertical Asymptote (s) : $x=2$

Horizontal Asymptote (s) : $y=1$

Range : $\{y \in \mathbb{R},\ y \neq 1\}$

$y=\dfrac{x-3}{x-1}$



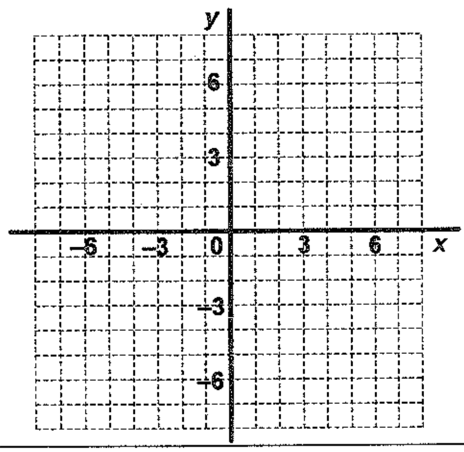
Domain : $\{x \in \mathbb{R},\ x \neq 1\}$

Vertical Asymptote (s) : $x=1$

Horizontal Asymptote (s) : $y=1$

Range : $\{y \in \mathbb{R},\ y \neq 1\}$

$y=\dfrac{3x}{x-2}$



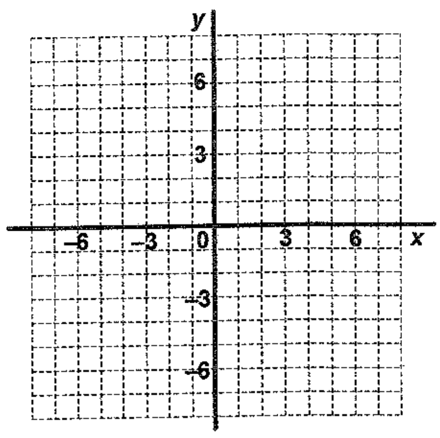
Domain : $\{x \in \mathbb{R},\ x \neq 2\}$

Vertical Asymptote (s) : $x=2$

Horizontal Asymptote (s) : $y=3$

Range : $\{y \in \mathbb{R},\ y \neq 3\}$

$y=\dfrac{6x-2}{2x-1}$



Domain : $\{x \in \mathbb{R},\ x \neq \frac{1}{2}\}$

Vertical Asymptote (s) : $x=\frac{1}{2}$

Horizontal Asymptote (s) : $y=3$

Range : $\{y \in \mathbb{R},\ y \neq 3\}$

For rational functions $f(x)=\dfrac{ax+b}{cx+d}$,

1. **Vertical asymptotes (may) occur where** $x = -d/c$ $(cx+d=0)$.
2. **Horizontal asymptotes occur where** $y = a/c$ (divide coefficients of x).

Why is the HA always a/c for a linear over linear function?

(think about what happens as $x \to \pm\infty$)

$$y = \frac{6x-2}{2x-1}$$

① $y(2x-1) = 6x-2$

$2xy - y = 6x-2$

$2xy - 6x = y-2$

$x(2y-6) = y-2$

$x = \dfrac{y-2}{2y-6}$

$2y-6 \neq 0$

$y \neq 3$

↑

HA

② $\boxed{\dfrac{K}{\infty} \to 0}$

$$y = \frac{6x-2}{2x-1}$$

divide by highest degree for x.

$$y = \frac{\frac{6x}{x} - \frac{2}{x}}{\frac{2x}{x} - \frac{1}{x}}$$

$$y = \frac{6 - \cancel{\frac{2}{x}}}{2 - \cancel{\frac{1}{x}}}$$

$y \to 3$

③ $y = \frac{6x-2}{2x-1}$

$$\begin{array}{r} 3 \\ 2x-1 \overline{)\ 6x-2} \\ -(6x-3) \\ \hline 1 \end{array}$$

$\therefore\ 6x-2 = 3(2x-1)+1$

$$y = \frac{6x-2}{2x-1} = \frac{3(2x-1)+1}{2x-1}$$

$$= \frac{3(2x-1)}{2x-1} + \frac{1}{2x-1}$$

$$y = 3 + \frac{1}{2x-1}$$

$$y = \frac{1}{2x-1}$$

$$y = \frac{\frac{1}{x}}{\frac{2x}{x} - \frac{1}{x}}$$

$$y = \frac{\frac{1}{x}}{2 - \frac{1}{x}}$$

$\therefore$ degree(num) < degree(denom.) then HA is $\boxed{y=0}$

degree(num) = degree(denom) then HA is $y = \frac{a}{c}$ ↑ leading coeff.

eg. $y = \frac{2x^3 - 5x^2 + 1}{5x^3 + 9}$

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