

**Definitions**

**Polynomial Expression:** A collection of terms, degree is positive integers

monomial :  $2x^2$ .

binomials :  $3x^2 + 1$

Trinomials :  $4x^3 - 2x - 9$ .

Fig. 1. Which of the following is a polynomial expression? Explain

	Yes/No	Reason
$5x^{\frac{1}{2}}$	✓	
$2x^{\ominus} + 3\sqrt{x} - 4$	✗	negative & fraction Exponent
$t^2 + 3.5t$	✓	
$3xy^2 + 4x^2$	✓	
$\frac{1}{2}x^5 - 3x^2 - x + 1$	✓	

**Polynomial Function:**

Type of Polynomial Functions:

Type	Degree	Standard Form $a \neq 0, a, b, c \dots \text{ are } \mathbb{R}$	Example
Constant.	0	$f(x) = ax^0$ $a \neq 0$	$f(x) = 5$ $y = 5$
linear	1	$f(x) = ax + b$ $a \neq 0$	$f(x) = -2x - 1$
quadratic.	2.	$f(x) = ax^2 + bx + c$	
Cubic	3	$f(x) = ax^3 + bx^2 + cx + d$	$f(x) = 3x^3 - 2x + 1$
quartic.	4	$f(x) = ax^4 + bx^3 + cx^2 + dx + e$	
$n$	$n$	$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$	

**Definitions of Polynomial Functions:**

- $a_n$  is the leading coefficient  $a_n \neq 0$ .
- $a_n, a_{n-1}, \dots, a_1, a_0 \in \mathbb{R}$ .
- all exponent are non-negative integers.
- degree is  $n$ .

Ex.2. Complete the following table.

Type	Degree	Standard Form <i>type.</i>	Example <i>leading coefficients</i>
$f(x) = 10x + 7$	1	linear	10
$f(x) = 3.7x^5 - 3x^2 - x + 0.1$	5	quintic, degree 5 poly F.	3.7
$f(x) = 2.5$	0	constant	2.5
$f(x) = -\frac{1}{2}x^2 + \sqrt{2}x - 5$	2	quadratic.	$-\frac{1}{2}$
$f(x) = x(x+1)(x-2)$	3	Cubic	1

Fig. 3. Which of the following is a polynomial <sup>function</sup> expression? Explain

	Yes/No	Reason
<del>5x</del> $f(x) = 5x$	✓	
$f(x) = 2x^{-3} + 3x - 4$	X	Negative
$f(x) = 2^x$	X	degree is a variable
$y = 3xz + 4x^2$	X	input $\rightarrow x$ , output $\rightarrow y$
$f(x) = \frac{1}{x-1}$	X	reciprocal function.

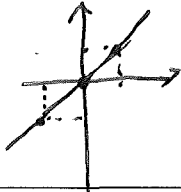
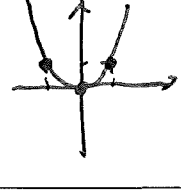
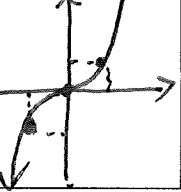
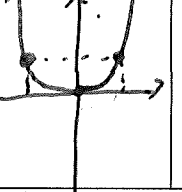
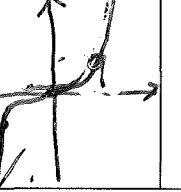
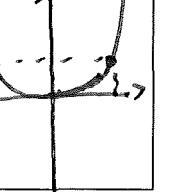
$= (x-1)^{-1}$

$= x^{-1}$

**Power Function:**

$$f(x) = a x^n \quad \begin{cases} a > 0 \\ a = 1 \rightarrow f(x) = x^n \\ a < 0 \end{cases}$$

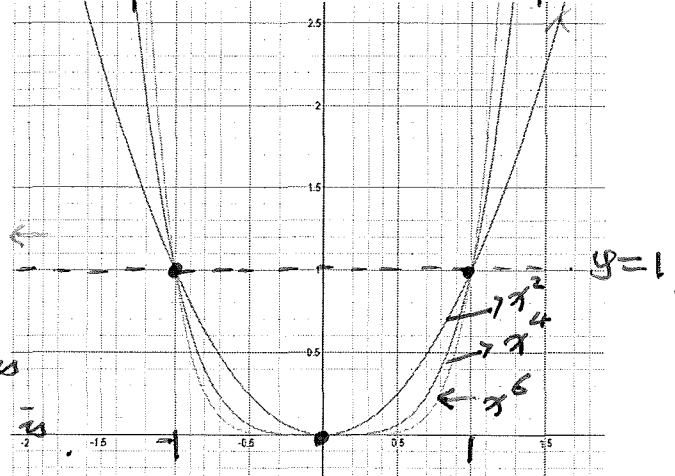
Complete the following chart  $a=1$

	$y = x$	$y = x^2$	$y = x^3$	$y = x^4$	$y = x^5$	$y = x^6$
<b>Degree</b>	1	2	3	4	5	6
<b>Name</b>	linear	quadratic	cubic	quartic	quintic	sextic
<b>Basic Shape (Sketch)</b>						

How can you tell the difference between  $y = x^2$ ,  $y = x^4$ , and  $y = x^6$  on a graph?

below  $y = 1$ , the higher exponents,  
 the wider the graph is,

above  $y = 1$ , the higher exponents,  
 the narrower the graph is  
 the steeper the slope is



**Things to take note of:**

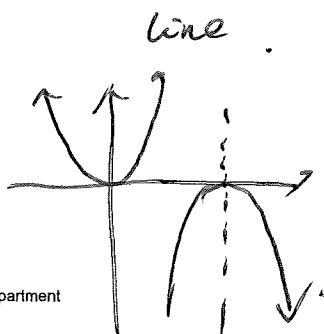
The **domain and range** of a function can be stated as an inequality or in interval notation.

Example:

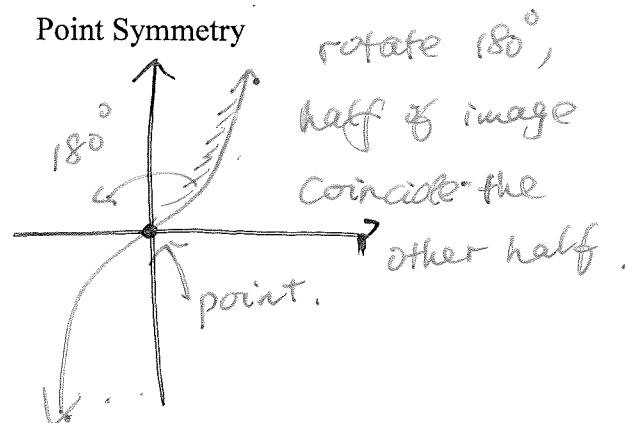
inclusive  $\rightarrow [ \text{smaller} , \text{bigger} ] \leftarrow$  exclude

When sketching a function it is useful to know if there is **symmetry**. There are two types of symmetry.

Line Symmetry *mirror image along a vertical line*

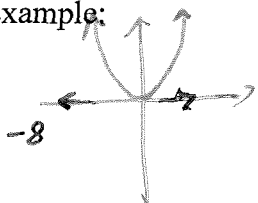


Point Symmetry



We will also discuss the **end behaviour** of a function. That is what happens to the  $y$  value of the function as the  $x$  approaches infinity or negative infinity. The mathematical notation for this is  $x \rightarrow \infty$  or  $x \rightarrow -\infty$ .

Example:



$$\begin{cases} \text{As } x \rightarrow \infty, y \rightarrow \infty \\ \text{As } x \rightarrow -\infty, y \rightarrow \infty \end{cases}$$

Complete the following chart

	Odd Power function, $y = x^n$ , $n$ is odd	Even Power function, $y = x^n$ , $n$ is even
<b>Domain</b>	$x \in \mathbb{R}$	$x \in \mathbb{R}$
<b>Range</b>	$y \in \mathbb{R}$	$[0, \infty)$
<b>Symmetry</b>	point symmetry.	line symmetry.
<b>End Behaviour</b>	As $x \rightarrow \infty$ , $y \rightarrow \infty$ As $x \rightarrow -\infty$ , $y \rightarrow -\infty$	As $x \rightarrow \infty$ , $y \rightarrow \infty$ As $x \rightarrow -\infty$ , $y \rightarrow \infty$