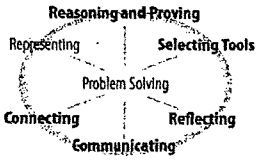
For help with questions 8 and 9, refer to Example 3.

8. A snowboard manufacturer determines that its profit, P, in thousands of dollars, can be modelled by the function $P(x) = x + 0.001\,25x^4 - 3$, where x represents the number, in hundreds, of snowboards sold.



a) What type of function is $P(x)$?

b) Without calculating, determine which finite differences are constant for this polynomial function. What is the value of the constant finite differences? Explain how you know.

c) Describe the end behaviour of this function, assuming that there are no restrictions on the domain.

d) State the restrictions on the domain in this situation.

e) What do the x-intercepts of the graph represent for this situation?

f) What is the profit from the sale of 3000 snowboards?

9. Use Technology The table shows the displacement, s, in metres, of an inner tube moving along a waterslide after time, t, in seconds.

t (s)	s (m)
0	10
1	34
2	42
3	46
4	58
5	90
6	154
7	262

a) Use finite differences to

i) identify the type of polynomial function that models s

ii) determine the value of the leading coefficient

b) *Graph the data in the table using a graphing* calculator. Use the regression feature of the graphing calculator to determine an equation for the function that models this situation.

10. a) Sketch graphs of $y = \sin x$ and $y = \cos x$.

b) Compare the graph of a periodic function to the graph of a polynomial function. Describe any similarities and differences. Refer to the end behaviour, local maximum and local minimum points, and maximum and minimum points.

11. The volume, V, in cubic centimetres, of a collection of open-topped boxes can be modelled by $V(x) = 4x^3 - 220x^2 + 2800x$, where x is the height of each box, in centimetres.

a) Graph $V(x)$. State the restrictions.

b) Fully factor $V(x)$. State the relationship between the factored form of the equation and the graph.

c) State the value of the constant finite differences for this function.

12. A medical researcher establishes that a patient's reaction time, r, in minutes, to a dose of a particular drug is $r(d) = -0.7d^3 + d^2$, where d is the amount of the drug, in millilitres, that is absorbed into the patient's blood.

a) What type of function is $r(d)$?

b) Without calculating the finite differences, state which finite differences are constant for this function. How do you know? What is the value of the constant differences?

c) Describe the end behaviour of this function if no restrictions are considered.

d) State the restrictions for this situation.

13. By analysing the impact of growing economic conditions, a demographer establishes that the predicted population, P, of a town t years from now can be modelled by the function $p(t) = 6t^4 - 5t^3 + 200t + 12\,000$.

a) Describe the key features of the graph represented by this function if no restrictions are considered.

b) *What is the value of the constant finite* differences?

c) What is the current population of the town?

d) What will the population of the town be 10 years from now?

e) When will the population of the town be approximately 175 000?

Achievement Check

14. Consider the function $f(x) = x^3 + 2x^2 - 5x - 6$.

a) How do the degree and the sign of the leading coefficient correspond to the end behaviour of the polynomial function?

b) Sketch a graph of the polynomial function.

c) What can you tell about the value of the third differences for this function?

Extend and Challenge

15. Graph a polynomial function that satisfies each description.

a) a quartic function with a negative leading coefficient and three x-intercepts

b) a cubic function with a positive leading coefficient and two x-intercepts

c) a quadratic function with a positive leading coefficient and no x-intercepts

d) a quintic function with a negative leading coefficient and five x-intercepts

16. a) What possible numbers of x-intercepts can a quintic function have?

b) Sketch an example of a graph of a quintic function for each possibility in part a).

17. Use Technology

a) What type of polynomial function is each of the following? Justify your answer.

i) $f(x) = (x + 4)(x - 1)(2x + 5)$

ii) $f(x) = (x + 4)^2(x - 1)$

iii) $f(x) = (x + 4)^3$

b) Graph each function.

c) Describe the relationship between the x-intercepts and the equation of the function.

18. A storage tank is to be constructed in the shape of a cylinder such that the ratio of the radius, r, to the height of the tank is $1:3$.

a) Write a polynomial function to represent

i) the surface area of the tank in terms of r

ii) the volume of the tank in terms of r

b) Describe the key features of the graph that corresponds to each of the above functions.

19. Math Contest

a) Given the function $f(x) = x^3 - 2x$, sketch $y = f(|x|)$.

b) Sketch $g(x) = |x^2 - 1| - |x^2 - 4|$.

c) Sketch the region in the plane to show all points (x, y) such that $|x| + |y| \leq 2$.

CAREER CONNECTION

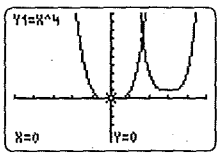
Davinder completed a 2-year course in mining engineering technology at a Canadian college. He works with an engineering and construction crew to blast openings in rock faces for road construction. In his job as the explosives specialist, he examines the structure of the rock in the blast area and determines the amounts and kinds of explosives needed to ensure that the blast is not only effective but also safe. He also considers environmental concerns such as vibration and noise. Davinder uses mathematical reasoning and a knowledge of physical principles to choose the correct formulas to solve problems. Davinder then creates a blast design and initiation sequence.

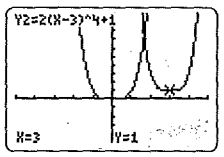
c) Answers may vary. Sample answer: $y = (-x)^{2n}$ has the same graph as $y = x^{2n}$, n is a non-negative integer, $(-x)^{2n} = (-1)^{2n}(x)^{2n} = x^{2n}$; $y = (-x)^{2n+1}$ has the same graph as $y = -x^{2n+1}$, n is a non-negative integer, $(-x)^{2n+1} = (-1)^{2n+1}(x)^{2n+1} = -x^{2n+1}$

15. a) Answers may vary. Sample answer: For the graph of $y = ax^n$, if $a > 0$, vertical stretch by a factor of a if $0 < a < 1$ vertical compression by a factor of a; if $-1 < a < 0$, vertical compression by a factor of a and a reflection in the x-axis; if $a < -1$, vertical stretch by a factor of a and a reflection in the x-axis

16. a) vertical stretch by a factor of 2, translation 3 units right, translation 1 unit up **b)** vertical stretch by a factor of 2, translation 3 units right, translation 1 unit up

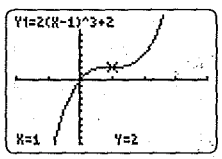
c)

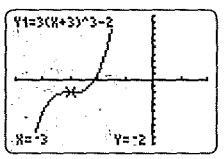




17. a) The second graph is a stretch (or compression) of factor a, a horizontal shift of units right, and a verticle shift of k units up

b) Answers may vary. Sample answers:





18. 124

19. $\left(4, \frac{8}{3}\right), \left(6, \frac{7}{3}\right)$

1.2 Characteristics of Polynomial Functions, pages 26–29

1. a) 4 **b)** 5 **c)** 4 **d)** 5 **e)** 3

2. a)–d)

	Sign of Leading Coefficient	End Behaviour (quadrants)	Symmetry	Number of Local Maximum Points	Number of Local Minimum Points
a)	+	2 to 1	none	1	2
b)	+	3 to 1	none	2	2
c)	−	3 to 4	none	2	1
d)	−	2 to 4	none	2	2
e)	−	2 to 4	point	1	1

d) If the function has a minimum or maximum point, the degree of the function is even. If the function has no maximum or minimum point, the degree is odd. The number of local maximums and local minimums is less than or equal to the degree of the function minus one.

3.

	i) End Behaviour (quadrants)	ii) Constant Finite Differences	iii) Value of Constant Finite Differences
a)	2 to 1	2nd	2
b)	2 to 4	3rd	−24
c)	3 to 4	4th	−168
d)	3 to 1	5th	72
e)	2 to 4	1st	−1
f)	3 to 4	6th	−720

4. a) 2, −4 **b)** 4, −2 **c)** 3, −2 **d)** 4, 1 **e)** 3, 6 **f)** 5, $\frac{1}{2}$

5. a) odd **b)** even **c)** odd **d)** even

6.

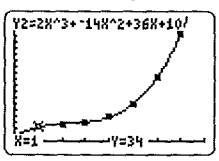
Graph	a) Least Degree	b) Sign of Leading Coefficient	c) End Behaviour (quadrants)	d) Symmetry
5a)	5	−	2 to 4	point
5b)	4	+	2 to 1	line
5c)	3	+	3 to 1	point
5d)	6	−	3 to 4	none

7. a) i) 3 **ii)** + **iii)** 1 **b) i)** 4 **ii)** − **iii)** −1

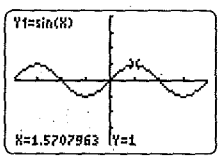
8. a) quartic **b)** fourth, 0.03 **c)** quadrant 2 to quadrant 1 **d)** $x \geq 0$ **e)** Answers may vary. Sample answer: They represent when the profit is equal to zero. **f)** $1 039 500

9. a) i) cubic (degree 3) **ii)** 2

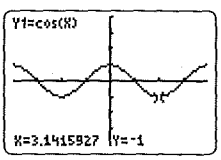
b)



10. a) i)

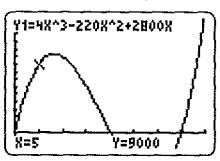


ii)



b) Answers may vary.

11. a) $x \geq 0$, $V(x) \geq 0$



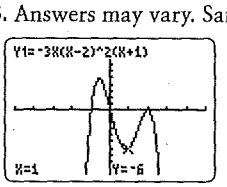
b) $V(x) = 4x(x - 35)(x - 20)$; x-intercepts 35, 20, 0 **c)** 24

12. a) cubic **b)** third, −4.2 **c)** quadrant 2 to 4 **d)** $\{d \in \mathbb{R}, d \geq 0\}$, $\{r \in \mathbb{R}, r \geq 0\}$

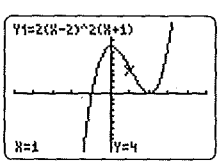
13. a) Answers may vary. Sample answer: quadrant 2 to quadrant 1, $\{x \in \mathbb{R}\}$, $\{P(t) \in \mathbb{R}, P(t) \geq 11\,732\}$, no x-intercepts **b)** 144 **c)** 12 000 **d)** 69 000 **e)** 13 years

15. Answers may vary. Sample answers:

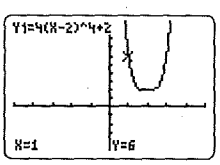
a)



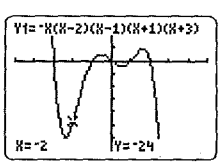
b)



c)

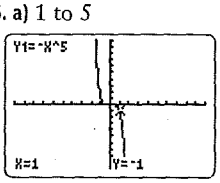


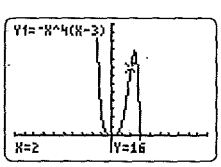
d)

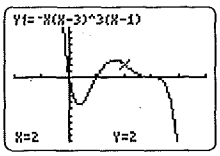


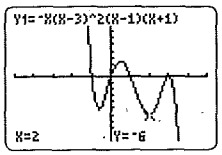
16. a) 1 to 5

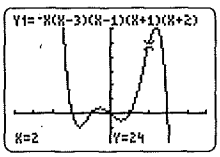
b)



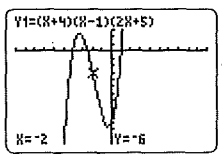


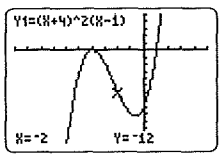


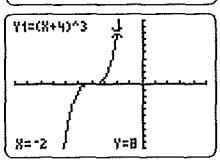




17. a) *i)* cubic *ii)* cubic *iii)* cubic

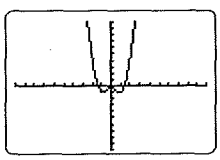
b)


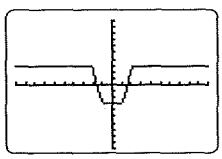


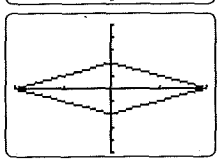


c) Answers may vary. Sample answer: The number of x-intercepts equals the number of roots of the equation.

18. a) *i)* $S(r) = 8\pi r^2$ *ii)* $V(r) = 3\pi r^3$ **b)** Answers may vary. Sample answer: $S(r)$ quadratic, has one x-intercept, $\{x \in \mathbb{R}, x \geq 0\}$, $\{y \in \mathbb{R}, y \geq 0\}$, from quadrant 2 to quadrant 1; $V(r)$ cubic, one x-intercept, $\{x \in \mathbb{R}\}$, $\{V \in \mathbb{R}\}$ from quadrant 3 to quadrant 1

19. a)


b)


c)


1.3 Equations and Graphs of Polynomial Functions, pages 39–41

1. a) *i)* 3, + *ii)* quadrant 3 to quadrant 1 *iii)* 4, -3, $\frac{1}{2}$

b) *i)* 4, − *ii)* quadrant 3 to quadrant 4 *iii)* -2, 2, 1, -1

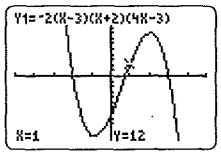
c) *i)* 5, + *ii)* quadrant 3 to quadrant 1 *iii)* $-\frac{2}{3}$, 4, -1, $\frac{3}{2}$

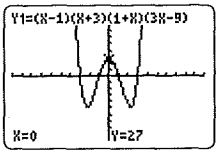
d) *i)* 6, − *ii)* quadrant 3 to quadrant 4 *iii)* -5, 5

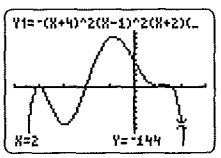
2. a) *i)* -4, $-\frac{1}{2}$, 1 *ii)* positive, $-4 < x < -\frac{1}{2}$, $x > 1$; negative $x < -4$, $-\frac{1}{2} < x < 1$ *iii)* no zeros of order 2 or 3

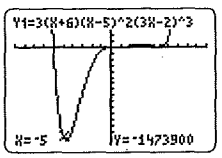
b) *i)* -1, 4 *ii)* negative $x < -1$, $-1 < x < 4$, $x > 4$ *iii)* could have zeros of order 2 **c)** *i)* -3, 1 *ii)* positive $x < -3$, $x > 1$; negative $-3 < x < 1$ *iii)* could have zeros of order 3 **d)** *i)* -5, 3 *ii)* positive $x < -5$, $-5 < x < 3$; negative $x > 3$ *iii)* could have zeros of order 2

3. a) *i)* -2, 3, $\frac{3}{4}$, all order 1 *ii)* -3, -1, 1, 3, all order 1 *iii)* order 2: -4, 1; order 1: -2, $\frac{3}{2}$ *iv)* order 3: $\frac{2}{3}$; order 2: 5; order 1: -6 **b)** graph in part ii) is even; others are neither

c) *i)*


ii)


iii)


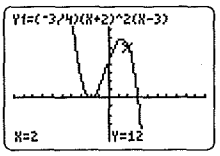
iv)


4. b) d) line, even; these functions have line symmetry about the y-axis because they are even functions. **a) c)** neither, neither; there is no symmetry about the origin or about the y-axis because these functions are neither even nor odd.

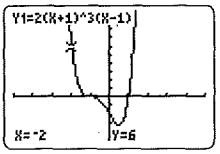
5. a) *i)* even *ii)* line **b)** *i)* odd *ii)* point **c)** *i)* neither *ii)* neither **d)** *i)* neither *ii)* neither **e)** *i)* even *ii)* line

6. a) $y = (x + 2)^3(x - 3)^2$ **b)** $y = -2(x + 3)(x + 1)(x - 2)$ **c)** $y = -3(x + 2)^2(x - 1)^2$ **d)** $y = 0.5(x + 2)^3(x - 1)^2$

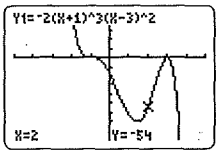
7. a) $y = -\frac{3}{4}(x + 2)^2(x - 3)$, neither



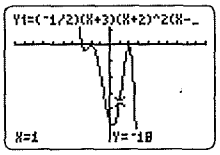
b) $y = 2(x + 1)^3(x - 1)$, neither



c) $y = -2(x + 1)^3(x - 3)^2$, neither

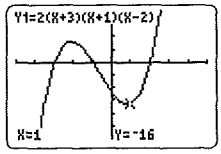


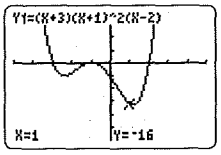
d) $y = -\frac{1}{2}(x + 3)(x + 2)^2(x - 2)^2$, neither

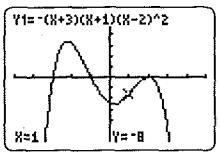


8. a) point **b)** line **c)** point **d)** point

9. Answers may vary. Sample answers:

a)


b)


c)


d)