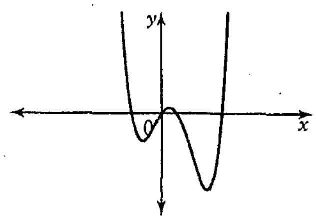
## Practise

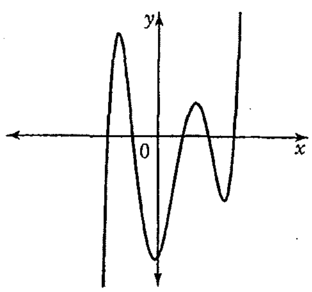
***For help with questions 1 to 3, refer to Example 1.***

**1.** Each graph represents a polynomial function of degree 3, 4, 5, or 6. Determine the least possible degree of the function corresponding to each graph. Justify your answer.

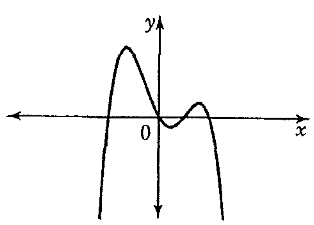
**a)**



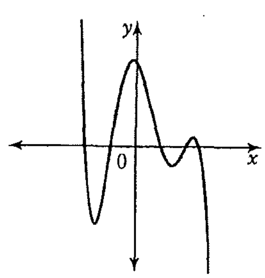

**b)**



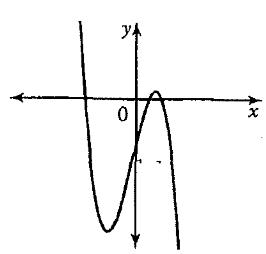

**c)**




**d)**

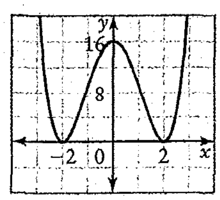


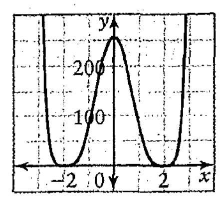

**e)**




CONNECTIONS

The least possible degree refers to the fact that it is possible for the graphs of two polynomial functions with either odd degree or even degree to *appear* to be similar, even though one may have a higher degree than the other. For instance, the graphs of $y = (x-2)^2(x+2)^2$ and $y = (x-2)^4(x+2)^4$ have the same shape and the same $x$-intercepts, −2 and 2, but one function has a double root at each of these values, while the other has a quadruple root at each of these values.




**2.** Refer to question 1. For each graph, do the following.

**a)** State the sign of the leading coefficient. Justify your answer.

**b)** Describe the end behaviour.

**c)** Identify any symmetry.

**d)** State the number of minimum and maximum points and local minimum and local maximum points. How are these related to the degree of the function?

**3.** Use the degree and the sign of the leading coefficient to

**i)** describe the end behaviour of each polynomial function

**ii)** state which finite differences will be constant

**iii)** determine the value of the constant finite differences

**a)** $f(x) = x^2 + 3x - 1$

**b)** $g(x) = -4x^3 + 2x^2 - x + 5$

**c)** $h(x) = -7x^4 + 2x^3 - 3x^2 + 4$

**d)** $p(x) = 0.6x^5 - 2x^4 + 8x$

**e)** $f(x) = 3 - x$

**f)** $h(x) = -x^6 + 8x^3$

*For help with question 4, refer to Example 2.*

4. State the degree of the polynomial function that corresponds to each constant finite difference. Determine the value of the leading coefficient for each polynomial function.

   **a)** second differences $= -8$

   **b)** fourth differences $= -48$

   **c)** third differences $= -12$

   **d)** fourth differences $= 24$

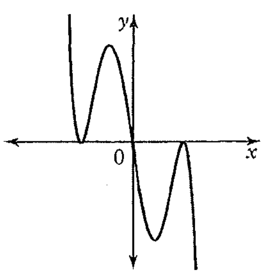
   **e)** third differences $= 36$

   **f)** fifth differences $= 60$

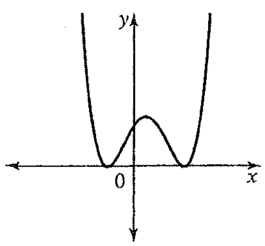
## Connect and Apply

5. Determine whether each graph represents an even-degree or an odd-degree polynomial function. Explain your reasoning.

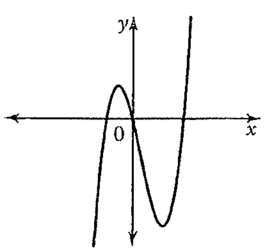
   **a)**



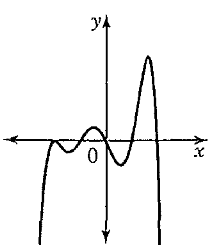

   **b)**




   **c)**




   **d)**




6. Refer to question 5. For each graph, do the following.

   **a)** State the least possible degree.

   **b)** State the sign of the leading coefficient.

   **c)** Describe the end behaviour of the graph.

   **d)** Identify the type of symmetry, if it exists.

*For help with question 7, refer to Example 2.*

7. Each table represents a polynomial function. Use finite differences to determine the following for each polynomial function.

   **i)** the degree

   **ii)** the sign of the leading coefficient

   **iii)** the value of the leading coefficient

   **a)**

| $x$ | $y$ |
|---|---|
| −3 | −45 |
| −2 | −16 |
| −1 | −3 |
| 0 | 0 |
| 1 | −1 |
| 2 | 0 |
| 3 | 9 |
| 4 | 32 |

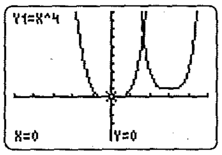
   **b)**

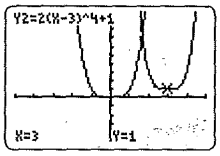
| $x$ | $y$ |
|---|---|
| −2 | −40 |
| −1 | 12 |
| 0 | 20 |
| 1 | 26 |
| 2 | 48 |
| 3 | 80 |
| 4 | 92 |
| 5 | 30 |

**c)** Answers may vary. Sample answer: $y = (-x)^{2n}$ has the same graph as $y = x^{2n}$, $n$ is a non-negative integer, $(-x)^{2n} = (-1)^{2n}(x)^{2n} = x^{2n}$; $y = (-x)^{2n+1}$ has the same graph as $y = -x^{2n+1}$, $n$ is a non-negative integer, $(-x)^{2n+1} = (-1)^{2n+1}(x)^{2n+1} = -x^{2n+1}$

**15. a)** Answers may vary. Sample answer: For the graph of $y = ax^n$, if $a > 0$, vertical stretch by a factor of $a$ if $0 < a < 1$ vertical compression by a factor of $a$; if $-1 < a < 0$, vertical compression by a factor of $a$ and a reflection in the $x$-axis; if $a < -1$, vertical stretch by a factor of $a$ and a reflection in the $x$-axis

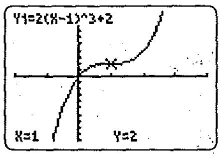
**16. a)** vertical stretch by a factor of 2, translation 3 units right, translation 1 unit up **b)** vertical stretch by a factor of 2, translation 3 units right, translation 1 unit up

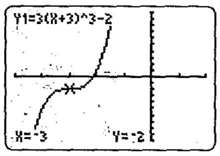
**c)** 




**17. a)** The second graph is a stretch (or compression) of factor $a$, a horizontal shift of units right, and a verticle shift of $k$ units up

**b)** Answers may vary. Sample answers:






**18.** 124

**19.** $\left(4, \frac{8}{3}\right), \left(6, \frac{7}{3}\right)$

## 1.2 Characteristics of Polynomial Functions, pages 26–29

**1. a)** 4 **b)** 5 **c)** 4 **d)** 5 **e)** 3

**2. a)–d)**

| | Sign of Leading Coefficient | End Behaviour (quadrants) | Symmetry | Number of Local Maximum Points | Number of Local Minimum Points |
|---|---|---|---|---|---|
| a) | + | 2 to 1 | none | 1 | 2 |
| b) | + | 3 to 1 | none | 2 | 2 |
| c) | − | 3 to 4 | none | 2 | 1 |
| d) | − | 2 to 4 | none | 2 | 2 |
| e) | − | 2 to 4 | point | 1 | 1 |

**d)** If the function has a minimum or maximum point, the degree of the function is even. If the function has no maximum or minimum point, the degree is odd. The number of local maximums and local minimums is less than or equal to the degree of the function minus one.

**3.**

| | i) End Behaviour (quadrants) | ii) Constant Finite Differences | iii) Value of Constant Finite Differences |
|---|---|---|---|
| a) | 2 to 1 | 2nd | 2 |
| b) | 2 to 4 | 3rd | −24 |
| c) | 3 to 4 | 4th | −168 |
| d) | 3 to 1 | 5th | 72 |
| e) | 2 to 4 | 1st | −1 |
| f) | 3 to 4 | 6th | −720 |

**4. a)** 2, −4 **b)** 4, −2 **c)** 3, −2 **d)** 4, 1 **e)** 3, 6 **f)** 5, $\frac{1}{2}$

**5. a)** odd **b)** even **c)** odd **d)** even

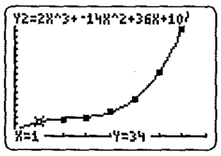
**6.**

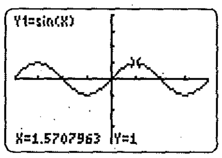
| Graph | a) Least Degree | b) Sign of Leading Coefficient | c) End Behaviour (quadrants) | d) Symmetry |
|---|---|---|---|---|
| 5a) | 5 | − | 2 to 4 | point |
| 5b) | 4 | + | 2 to 1 | line |
| 5c) | 3 | + | 3 to 1 | point |
| 5d) | 6 | − | 3 to 4 | none |

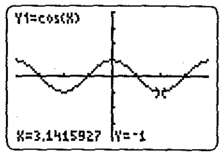
**7. a) i)** 3 **ii)** + **iii)** 1 **b) i)** 4 **ii)** − **iii)** −1

**8. a)** quartic **b)** fourth, 0.03 **c)** quadrant 2 to quadrant 1 **d)** $x \geq 0$ **e)** Answers may vary. Sample answer: They represent when the profit is equal to zero. **f)** \$1 039 500

**9. a) i)** cubic (degree 3) **ii)** 2

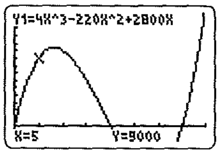
**b)** 


**10. a) i)** 

**ii)** 


**b)** Answers may vary.

**11. a)** $x \geq 0$, $V(x) \geq 0$

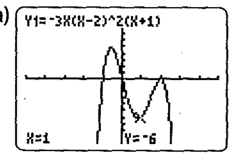


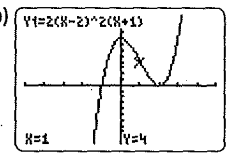

**b)** $V(x) = 4x(x - 35)(x - 20)$; $x$-intercepts 35, 20, 0 **c)** 24

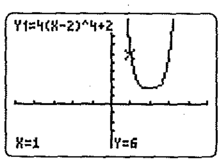
**12. a)** cubic **b)** third, −4.2 **c)** quadrant 2 to 4 **d)** $\{d \in \mathbb{R}, d \geq 0\}$, $\{r \in \mathbb{R}, r \geq 0\}$

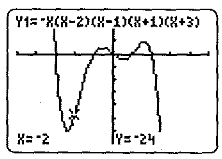
**13. a)** Answers may vary. Sample answer: quadrant 2 to quadrant 1, $\{x \in \mathbb{R}\}$, $\{P(t) \in \mathbb{R}, P(t) \geq 11\ 732\}$, no $x$-intercepts **b)** 144 **c)** 12 000 **d)** 69 000 **e)** 13 years

**15.** Answers may vary. Sample answers:

**a)** 

**b)** 


**c)** 

**d)** 


**16. a)** 1 to 5

**b)**