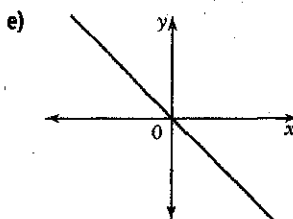
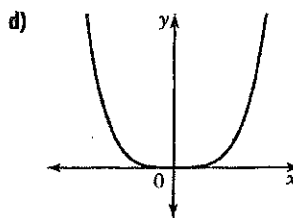
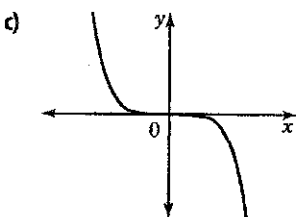
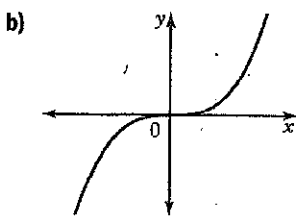
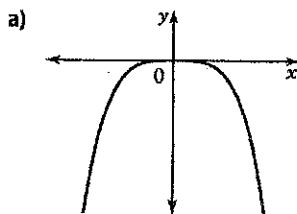


For help with question 3, refer to Example 2.

3. Consider each graph.

- Does it represent a power function of even degree? odd degree? Explain.
- State the sign of the leading coefficient. Justify your answer.
- State the domain and range.
- Identify any symmetry.
- Describe the end behaviour.



For help with question 4, refer to Example 3.

4. Copy and complete the table for the following functions.

$$\begin{array}{lll}
 y = -x^3 & y = \frac{3}{7}x^2 & y = 5x \\
 y = 4x^5 & y = -x^6 & y = -0.1x^{11} \\
 y = 2x^4 & y = -9x^{10} & 
 \end{array}$$

End Behaviour	Function	Reasons
Extends from quadrant 3 to quadrant 1		
Extends from quadrant 2 to quadrant 4		
Extends from quadrant 2 to quadrant 1		
Extends from quadrant 3 to quadrant 4		

### Connect and Apply

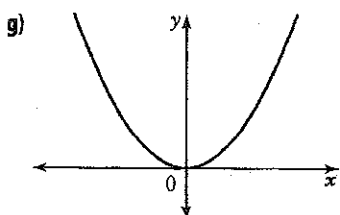
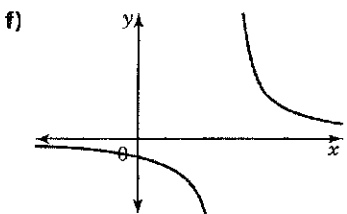
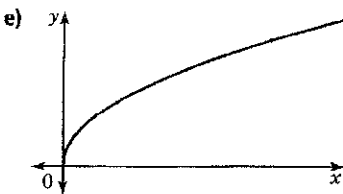
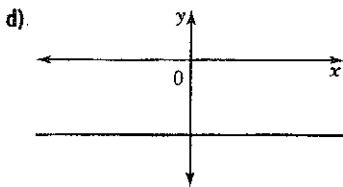
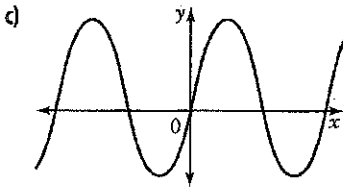
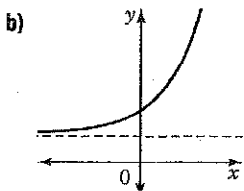
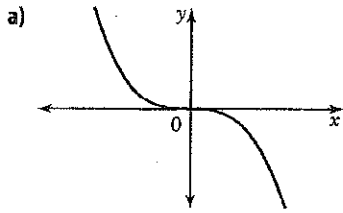
For help with questions 5 and 6, refer to Example 4.

- As a tropical storm intensifies and reaches hurricane status, it takes on a circular shape that expands outward from the eye of the storm. The area,  $A$ , in square kilometres, spanned by a storm with radius,  $r$ , in kilometres, can be modelled by the function  $A(r) = \pi r^2$ .
  - Graph  $A(r)$  for  $r \in [0, 10]$ .
  - State the domain and range.
  - Describe the similarities and differences between the graph of  $A(r)$  and the graph of  $y = x^2$ .
- The circumference,  $C$ , in kilometres, of the tropical storm in question 5 can be modelled by the function  $C(r) = 2\pi r$ .
  - Graph  $C(r)$  for  $r \in [0, 10]$ .
  - State the domain and range.
  - Describe the similarities and differences between the graph of  $C(r)$  and the graph of  $y = x$ .

7. Determine whether each graph represents a power function, an exponential function, a periodic function, or none of these. Justify your choice.

### CONNECTIONS

You worked with periodic functions when you studied trigonometric functions in grade 11. Periodic functions repeat at regular intervals.



### 8. Use Technology

- a) Graph  $f(x) = x^3 + x^2$ ,  $g(x) = x^3 - x$ , and  $h(x) = x^3$  on the same set of axes.



- b) Compare and describe the key features of the graphs of these functions.

### 9. Use Technology

- a) Graph  $f(x) = x^4 + x$ ,  $g(x) = x^4 - x^2$ , and  $h(x) = x^4$  on the same set of axes.

- b) Compare and describe the key features of the graphs of these functions.

10. Describe the similarities and differences between the line  $y = x$  and power functions with odd degree greater than one. Use graphs to support your answer.

11. Describe the similarities and differences between the parabola  $y = x^2$  and power functions with even degree greater than two. Use graphs to support your answer.

### 14. Use Technology

- a) Graph each pair of functions. What do you notice? Provide an algebraic explanation for what you observe.

i)  $y = (-x)^2$  and  $y = x^2$

ii)  $y = (-x)^4$  and  $y = x^4$

iii)  $y = (-x)^6$  and  $y = x^6$

- b) Repeat part a) for each of the following pairs of functions.

i)  $y = (-x)^3$  and  $y = -x^3$

ii)  $y = (-x)^5$  and  $y = -x^5$

iii)  $y = (-x)^7$  and  $y = -x^7$

- c) Describe what you have learned about functions of the form  $y = (-x)^n$ , where  $n$  is a non-negative integer. Support your answer with examples.

16. a) Describe the relationship between the graph of  $y = x^2$  and the graph of  $y = 2(x - 3)^2 + 1$ .

- b) Predict the relationship between the graph of  $y = x^4$  and the graph of  $y = 2(x - 3)^4 + 1$ .

- c) Verify the accuracy of your prediction by sketching the graphs in part b).

17. a) Use the results of question 16 to predict a relationship between the graph of  $y = x^3$  and the graph of  $y = a(x - b)^3 + k$ .

- b) Verify the accuracy of your prediction in part a) by sketching two functions of the form  $y = a(x - b)^3 + k$ .

18. Math Contest Determine the number of digits in the expansion of  $(2^{120})(5^{125})$  without using a calculator or computer.

# Answers

## CHAPTER 1

### Prerequisite Skills, pages 2-3

1. a) 7 b) -5 c) 11 d) 5 e)  $8x + 7$  f)  $-12x + 7$   
 2. a) 1 b) 10 c) 6 d) 0 e)  $24x^2 - 18x + 3$  f)  $18x^2 - 9x + 1$   
 3. a)  $m = 3, b = 2$  b)  $m = -\frac{1}{2}, b = \frac{3}{2}$  c)  $m = 5, b = 7$   
 d)  $m = -5, b = -11$  e)  $m = -\frac{1}{2}, b = 1$   
 4. a)  $y = 3x + 5$  b)  $y = 4x + 4$  c)  $y = -4x + 31$   
 d)  $y = -7x + 12$   
 5. a) linear b) neither c) quadratic  
 6. a)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq 1\}\}$  b)  $\{x \in \mathbb{R}, x \neq -5\}, \{y \in \mathbb{R}, y \neq 0\}$   
 c)  $\{x \in \mathbb{R}, x \leq \frac{1}{2}\}, \{y \in \mathbb{R}, y \geq 0\}$   
 7. Answers may vary. Sample answers:  
 a)  $y = -3(x + 1)(x - 1)$  b)  $y = -2x^2 - 3x + 3$   
 c)  $y = 4\left(x + \frac{1}{2}\right)(x - 2)$   
 8. a) x-intercepts  $-6, \frac{5}{2}$ ; vertex  $\left(-\frac{7}{4}, -\frac{289}{8}\right)$ ; opens up;  
 $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq -\frac{289}{8}\}\}$



- b) x-intercepts approximately 3.29, 4.71; vertex (4, 1); opens down;  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \leq 1\}\}$



- c) x-intercepts approximately -1.47, 7.47; vertex (3, 5); opens down;  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \leq 5\}\}$



- d) x-intercepts  $-2, \frac{3}{5}$ ; vertex  $\left(-\frac{7}{10}, -\frac{169}{20}\right)$ ; opens up;  
 $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \geq -\frac{169}{20}\}\}$



- e) x-intercepts  $1, \frac{2}{3}$ ; vertex  $\left(\frac{5}{6}, \frac{1}{12}\right)$ ; opens down;  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \leq \frac{1}{12}\}\}$



9. a) vertical stretch and a reflection in the x-axis b) vertical compression c) horizontal compression d) horizontal stretch and a reflection in the y-axis e) reflection in the y-axis  
 10. a) i)  $f(x) = x + 5$



- ii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}\}$   
 b) i)  $f(x) = -5(x + 1)^2 - 2$



- ii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \leq -2\}\}$

- c) i)  $f(x) = -3[2(x + 4)] + 6$



- ii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}\}$

11. a) i) vertical stretch by a factor of 2, reflection in the x-axis, translation 3 units left, translation 1 unit up  
 ii)  $y = -2(x + 3) + 1$  b) i) vertical compression by a factor of  $\frac{1}{3}$ , translation 2 units down ii)  $y = \frac{1}{3}x^2 - 2$

12. vertical stretch by a factor of 3, horizontal stretch by a factor of 2, reflection in the y-axis, translation 1 unit right, translation 2 units up

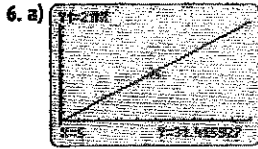
### 1.1 Power Functions, pages 11-14

1. a) No. b) Yes. c) Yes. d) Yes. e) No. f) No.  
 2. a) 4, 5 b) 1, -1 c) 2, 8 d) 3,  $-\frac{1}{4}$  e) 0, -5 f) 2, 1  
 3. a) i) even ii) negative iii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \leq 0\}\}$  iv) line  
 v) quadrant 3 to quadrant 4 b) i) odd ii) positive  
 iii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}\}$  iv) point v) quadrant 3 to quadrant 1  
 c) i) odd ii) negative iii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}\}$  iv) point  
 v) quadrant 2 to quadrant 4 d) i) even ii) negative  
 iii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}, y \leq 0\}\}$  iv) line v) quadrant 3 to quadrant 4  
 e) i) odd ii) negative iii)  $\{x \in \mathbb{R}, \{y \in \mathbb{R}\}\}$  iv) point  
 v) quadrant 2 to quadrant 4

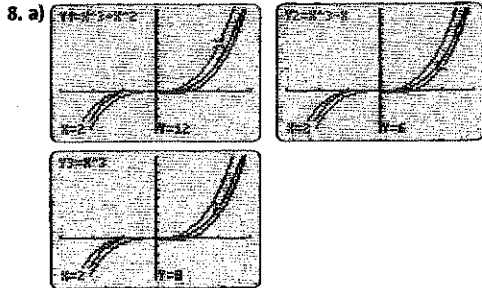
End Behaviour	Function
Extends from quadrant 3 to quadrant 1	$y = 5x, y = 4x^2$
Extends from quadrant 2 to quadrant 4	$y = -x^2, y = -0.1x^{17}$
Extends from quadrant 2 to quadrant 1	$y = \frac{3}{7}x^2, y = 2x^4$
Extends from quadrant 3 to quadrant 4	$y = -x^6, y = -9x^{10}$



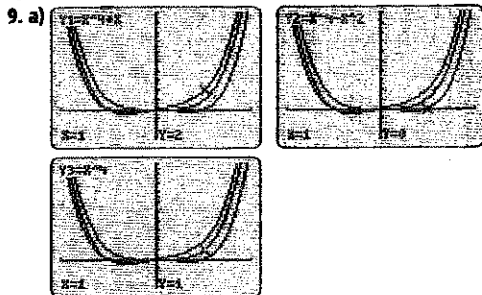
- b)  $\{r \in \mathbb{R}, 0 \leq r \leq 10\}; \{A \in \mathbb{R}, 0 \leq A \leq 100\pi\}$   
 c) Answers may vary. Sample answer: similarities—vertex (0, 0), x-intercept, y-intercept, end behaviour; differences—domain, range, shape



- b)  $\{r \in \mathbb{R}, 0 \leq r \leq 10\}$ ,  $\{C \in \mathbb{R}, 0 \leq C \leq 20\pi\}$   
 c) Answers may vary. Sample answer: similarities—end behaviour; differences—domain, range, shape  
 7. a) power (cubic) b) exponential c) periodic d) power (constant)  
 e) none of these f) none of these g) power (quadratic)

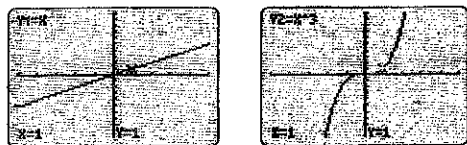


- b)  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}\}$ , quadrant 3 to quadrant 1, point symmetry about  $(0, 0)$ ; x-intercept 0, y-intercept 0



- b)  $\{x \in \mathbb{R}\}$ , quadrant 2 to quadrant 1; x-intercept 0, y-intercept 0

10. Answers may vary. Sample answer: similarities—extend from quadrant 1 to quadrant 3 (positive leading coefficient),  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}\}$ , point symmetry about  $(0, 0)$ ; differences—shape, extend from quadrant 2 to quadrant 4 (negative leading coefficient)



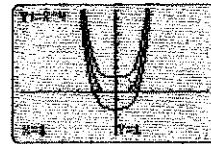
11. Answers may vary. Sample answer: similarities—extend from quadrant 2 to quadrant 1 (positive leading coefficient), domain, line symmetry; differences—shape, range, extend from quadrant 3 to quadrant 4 (negative leading coefficient)



12. a) Answers may vary. Sample answer: similarities—quadrant 3 to quadrant 1, domain, range, point symmetry, shape; difference—shifted vertically



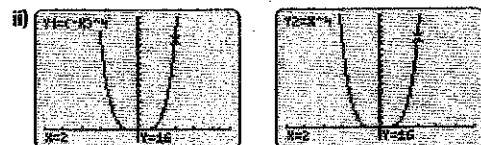
- b) Answers may vary. Sample answer: similarities—quadrant 2 to quadrant 1, domain, line symmetry, shape; differences—range, shifted vertically



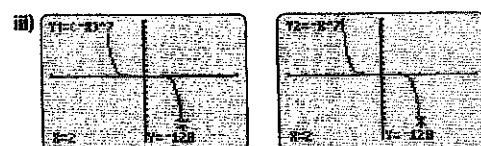
- c)  $c$  is a vertical shift of  $x^n$ ,  $n \in \mathbb{N}$

13. Answers may vary. Sample answer: path of a river:  $y = x^3$ ,  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}\}$ ; cross-section of a valley:  $y = x^2$ ,  $\{x \in \mathbb{R}\}$ ,  $\{y \in \mathbb{R}, y \geq 0\}$

14. a)  $y = (-x)^{2n}$  is the same graph as  $y = x^{2n}$ ,  $n$  is a non-negative integer,  $(-x)^{2n} = (-1)^{2n}(x)^{2n} = x^{2n}$



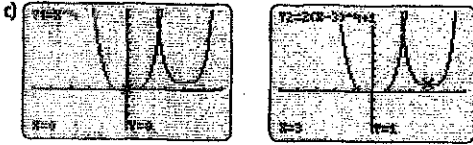
- b)  $y = (-x)^{2n+1}$  has the same graph as  $y = -x^{2n+1}$ ,  $n$  is a non-negative integer,  $(-x)^{2n+1} = (-1)^{2n+1}(x)^{2n+1} = -x^{2n+1}$



c) Answers may vary. Sample answer:  $y = (-x)^{2n}$  has the same graph as  $y = x^{2n}$ ,  $n$  is a non-negative integer,  $(-x)^{2n} = (-1)^{2n}(x)^{2n} = x^{2n}$ ;  $y = (-x)^{2n+1}$  has the same graph as  $y = -x^{2n+1}$ ,  $n$  is a non-negative integer,  $(-x)^{2n+1} = (-1)^{2n+1}(x)^{2n+1} = -x^{2n+1}$

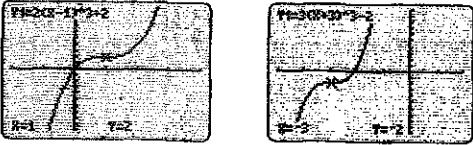
15. a) Answers may vary. Sample answer: For the graph of  $y = ax^n$ , if  $a > 0$ , vertical stretch by a factor of  $a$  if  $0 < a < 1$  vertical compression by a factor of  $a$ ; if  $1 < a < 0$ , vertical compression by a factor of  $a$  and a reflection in the  $x$ -axis; if  $a < -1$ , vertical stretch by a factor of  $a$  and a reflection in the  $x$ -axis

16. a) vertical stretch by a factor of 2, translation 3 units right, translation 1 unit up b) vertical stretch by a factor of 2, translation 3 units right, translation 1 unit up



17. a)  $a$  is a vertical stretch or compression;  $b$  is a shift left or right;  $k$  is a shift up or down

b) Answers may vary. Sample answers:



18. 124

19.  $(4, \frac{8}{3}), (6, \frac{7}{3})$

### 1.2 Characteristics of Polynomial Functions, pages 26–29

1. a) 4 b) 5 c) 4 d) 5 e) 3

2. a)–d)

Graph	Sign of Leading Coefficient	End Behaviour (quadrants)	Symmetry	Local Maximum Points (n)	Local Minimum Points (m)
1a)	+	2 to 1		2	1
1b)	+	3 to 1		2	2
1c)	-	3 to 4		1	2
1d)	-	2 to 4		2	2
1e)	-	2 to 4	point	1	1

d) If the function has a minimum or maximum point, the degree of the function is even. If the function has no maximum or minimum point, the degree is odd. The number of local maximums and local minimums is less than or equal to the degree of the function minus one.

	i) End Behaviour (quadrants)	ii) Constant Finite Differences	iii) Value of Constant Finite Differences
a)	2 to 1	2nd	2
b)	2 to 4	3rd	-24
c)	3 to 4	4th	-168
d)	3 to 1	5th	72
e)	2 to 4	1st	-1
f)	3 to 4	6th	-720

4. a) 2, -4 b) 4, -2 c) 3, -2 d) 4, 1 e) 3, 6 f) 5,  $\frac{1}{2}$

5. a) odd b) even c) odd d) even

6.

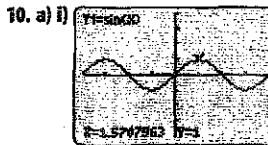
Graph	a) Least Degree	b) Sign of Leading Coefficient	c) End Behaviour (quadrants)	d) Symmetry
5a)	5	-	2 to 4	point
5b)	4	+	2 to 1	line
5c)	3	+	3 to 1	point
5d)	6	-	3 to 4	none

7. a) i) 3 ii) + iii) 1 b) i) 4 ii) - iii) -1

8. a) quartic b) fourth, 0.03 c) quadrant 2 to quadrant 1

d)  $x \geq 0$  e) Answers may vary. Sample answer: They represent when the profit is equal to zero. f) \$1 039 500

9. a) i) cubic (degree 3) ii) 2



b) Answers may vary.

11. a)  $x \geq 0, V(x) \geq 0$



b)  $V(x) = 4x(x - 35)(x - 20)$ ;  $x$ -intercepts 35, 20, 0 c) 24

12. a) cubic b) third, -4.2 c) quadrant 2 to 4

d)  $\{d \in \mathbb{R}, d \geq 0\}, \{r \in \mathbb{R}, r \geq 0\}$

13. a) Answers may vary. Sample answer: quadrant 2 to quadrant 1,  $\{x \in \mathbb{R}\}, \{P(t) \in \mathbb{R}, P(t) \geq 11\ 732\}$ , no  $x$ -intercepts

b) 144 c) 12 000 d) 69 000 e) 13 years

15. Answers may vary. Sample answers:



16. a) 1 to 5

