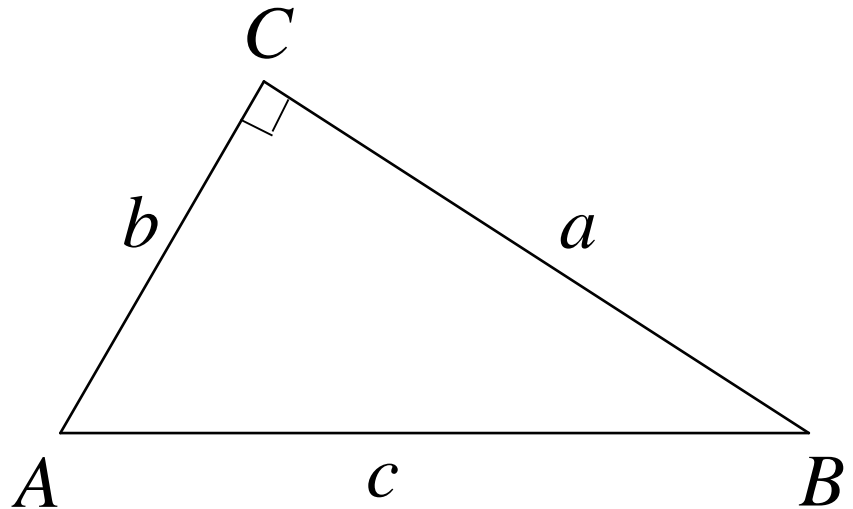


6.6 Adjusting the Pythagorean Theorem

The Cosine Law

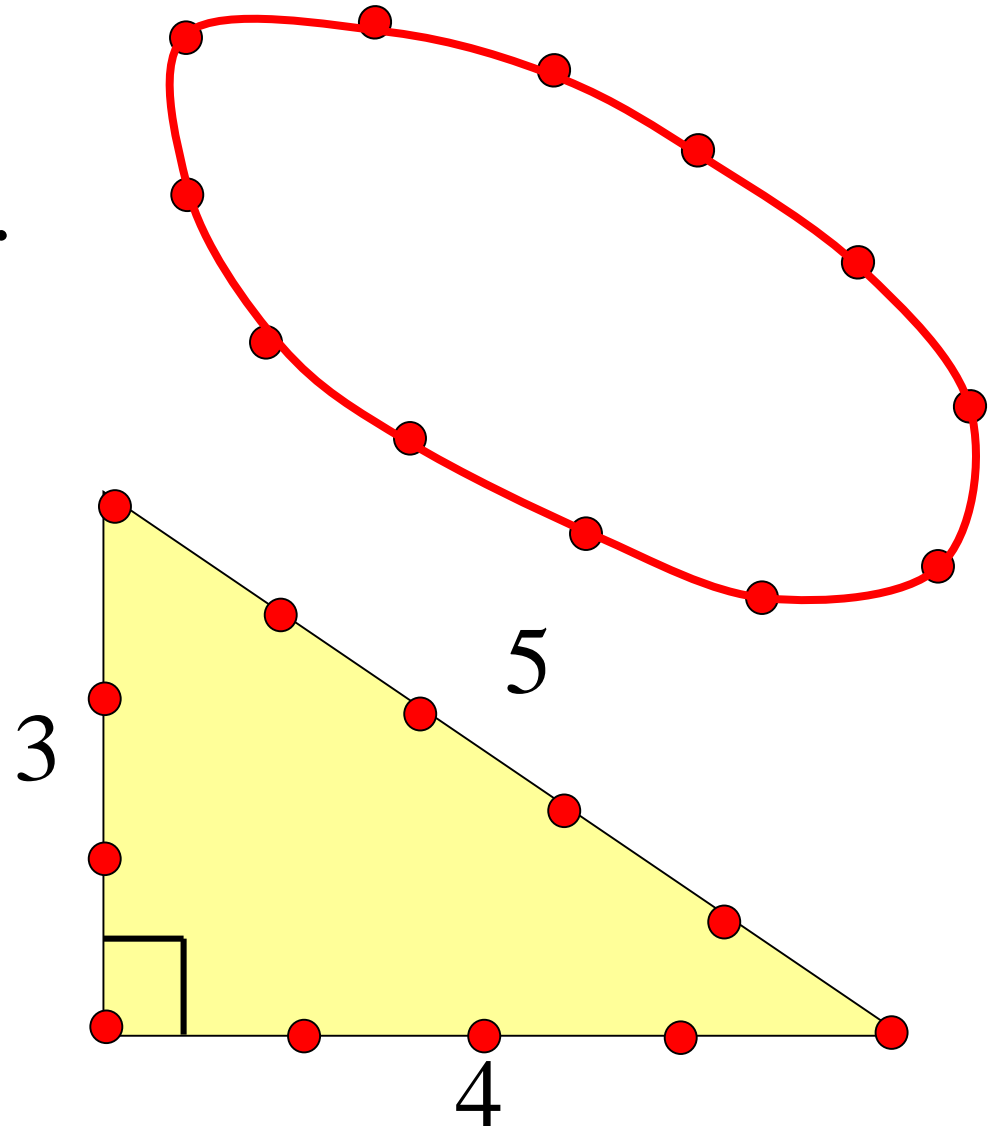
$$c^2 = a^2 + b^2$$



We start with a rope 12 units long. Units are marked.

We form the following figure.

A right angle is formed.



$3, 4, 5$ is a *Pythagorean Triple*

Which numbers are Pythagorean Triples?

a	b	c	
3	4	5	yes
5	6	7	no
5	12	13	yes

Try the Pythagorean Theorem

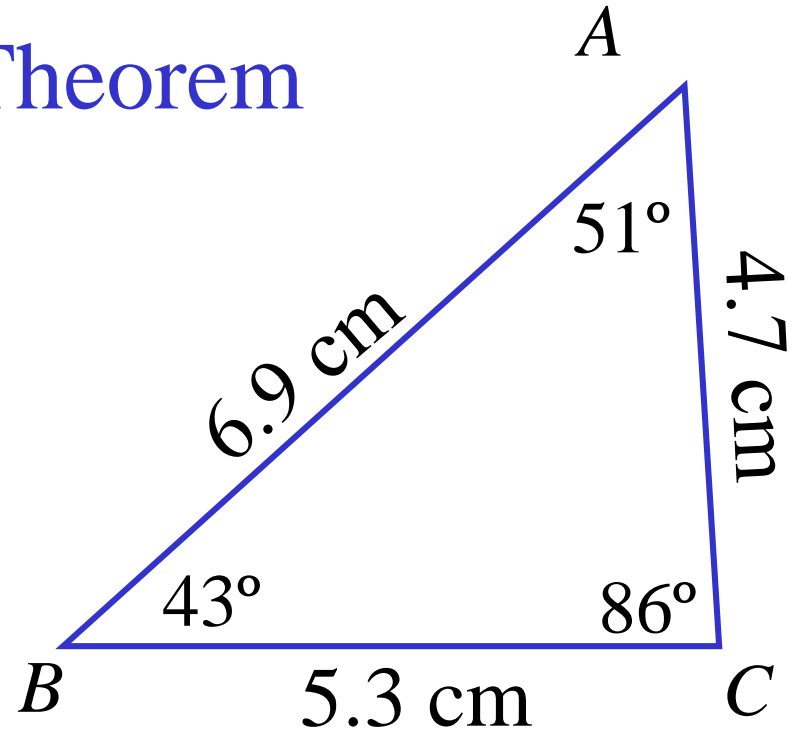
$$\angle C < 90^\circ$$

Compare c^2 with $a^2 + b^2$

$$c^2 \quad a^2 + b^2$$

$$6.9^2 \quad 5.3^2 + 4.7^2$$

$$c^2 < a^2 + b^2$$



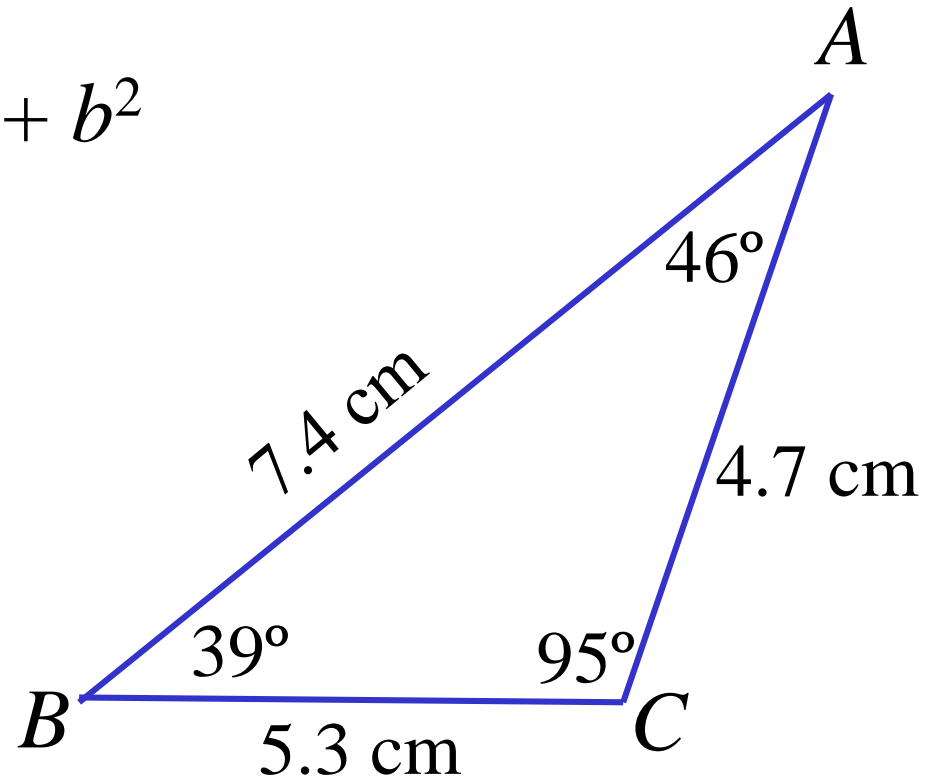
Try the Pythagorean Theorem $\angle C > 90^\circ$

Compare c^2 with $a^2 + b^2$

$$c^2 \quad a^2 + b^2$$

$$7.4^2 \quad 5.3^2 + 4.7^2$$

$$c^2 > a^2 + b^2$$

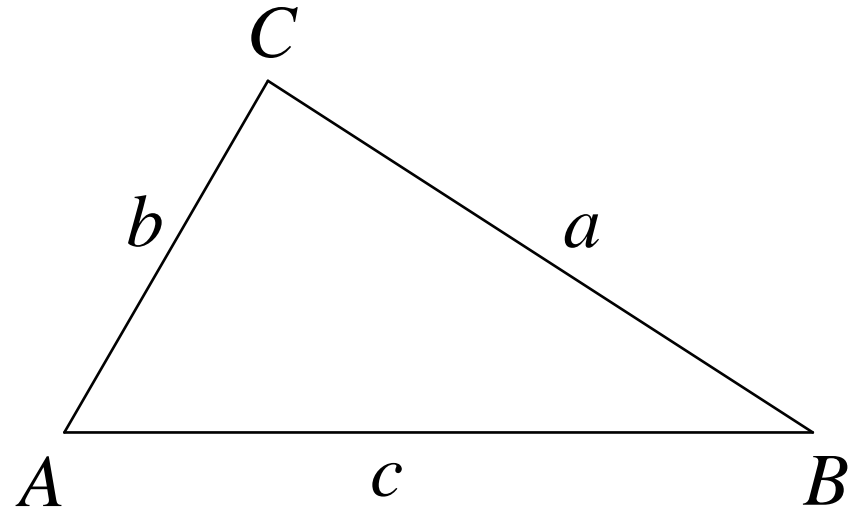


The Cosine Law

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$



The Cosine Law is used for situations involving **SAS** as well as **SSS**.

You are given 2 sides and the contained angle and you wish to find the third side **or** three side and you need to find one of the angles.

Example (1) Find a .

$$a^2 = b^2 + c^2 - 2bc \cos A$$

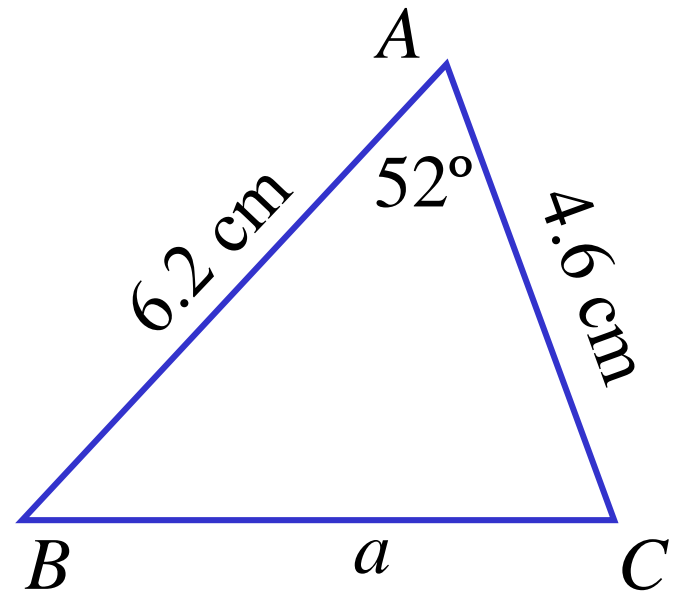
$$a^2 = 4.6^2 + 6.2^2 - 2(4.6)(6.2) \cos 52^\circ$$

$$a^2 = 21.16 + 38.44 - 35.12$$

$$a^2 = 24.48$$

$$a = \sqrt{24.48}$$

$$a = 4.9 \text{ cm}$$



Example (2) Find b .

$$b^2 = a^2 + c^2 - 2ac \cos B$$

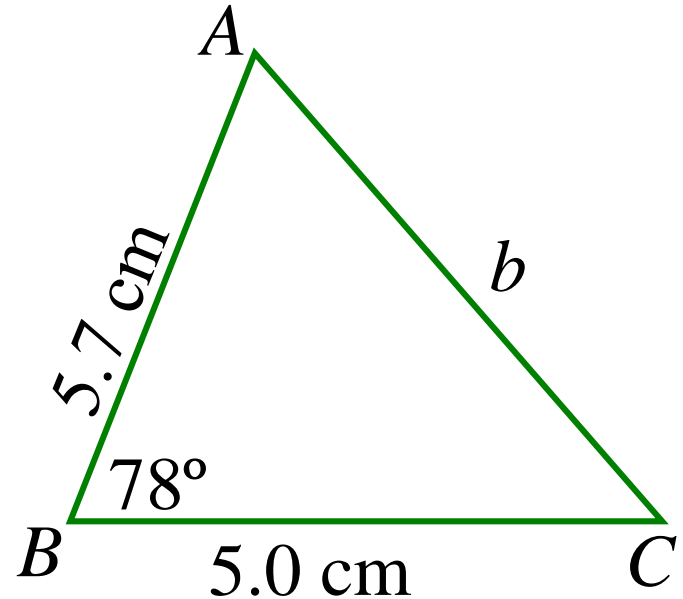
$$b^2 = 5.0^2 + 5.7^2 - 2(5.0)(5.7) \cos 78^\circ$$

$$b^2 = 25 + 32.49 - 11.85$$

$$b^2 = 45.64$$

$$b = \sqrt{45.64}$$

$$b = 6.8$$



Example (3) Find $\angle P$.

$$p^2 = q^2 + r^2 - 2qr \cos P$$

$$7^2 = 8^2 + 9^2 - 2(8)(9) \cos P$$

$$49 = 64 + 81 - 144 \cos P$$

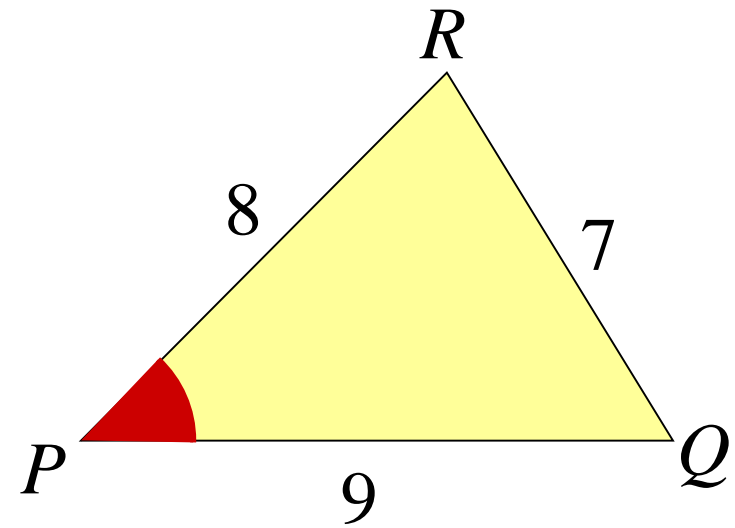
$$49 - 64 - 81 = -144 \cos P$$

$$-96 = -144 \cos P$$

$$\frac{-96}{-144} = \cos P$$

$$0.6666666 = \cos P$$

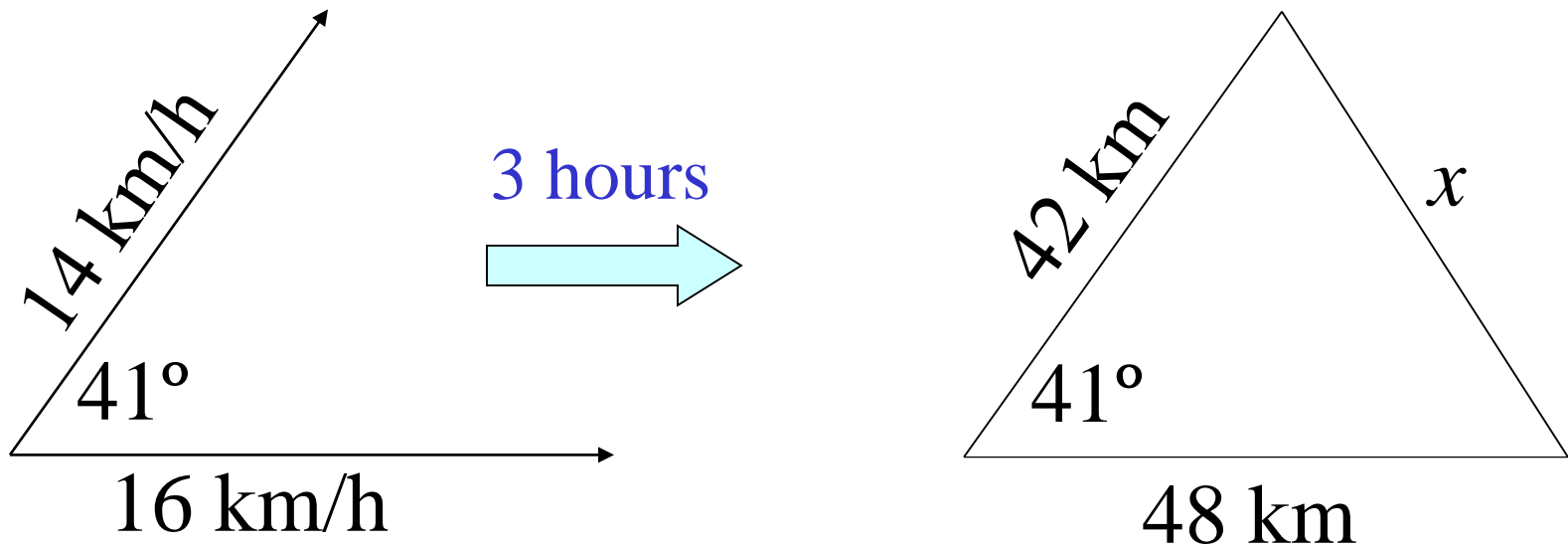
$$\angle P = \cos^{-1}(0.666666)$$



$$\angle P = 48.2^\circ$$

Example 4

Two girls begin cycling from the same location. The angle of the roads is 41° . One girl is cycling at 14 km/h and the second girl is cycling at 16 km/h. How far apart are the girls after 3 hours?



$$x^2 = 42^2 + 48^2 - 2(42)(48)\cos 41^\circ$$

$$x^2 = 1025$$

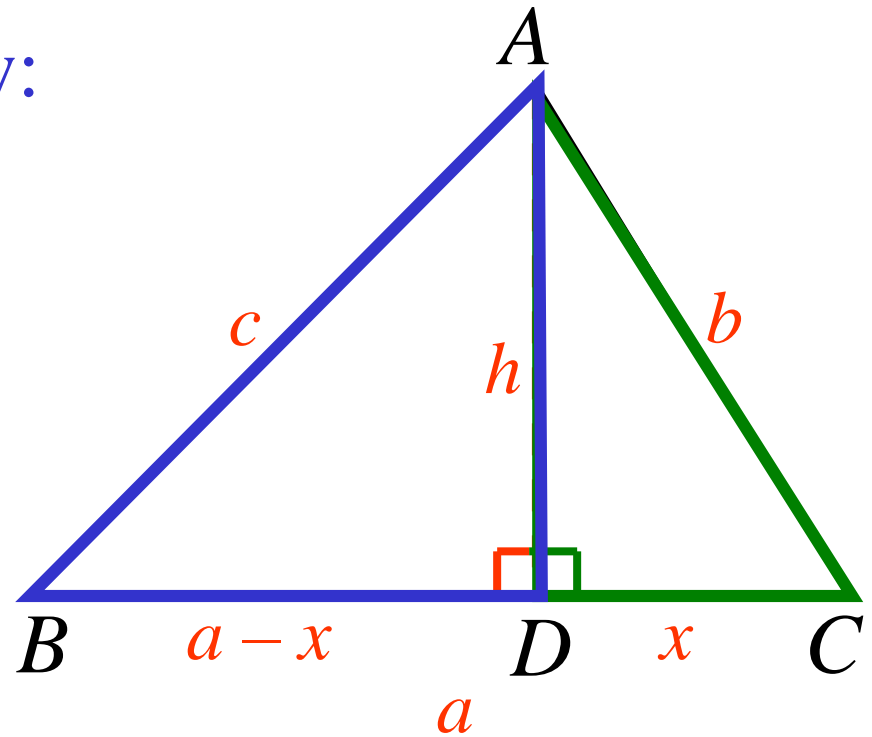
$$x = \sqrt{1025}$$

$$x = 32 \text{ km}$$

Deriving the Cosine Law:

Given $\triangle ABC$ and all angles are acute.

Draw altitude AD .



In $\triangle ADC$

$$\frac{x}{b} = \cos C$$

$$x = b \cos C$$

and $b^2 = x^2 + h^2$

In $\triangle ABD$

$$c^2 = h^2 + (a-x)^2$$

$$c^2 = h^2 + a^2 - 2ax + x^2$$

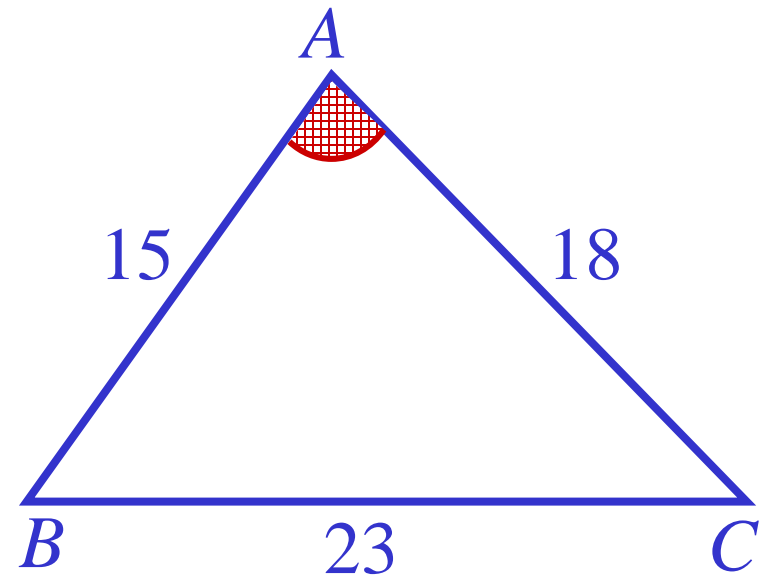
$$c^2 = a^2 + x^2 + h^2 - 2ax$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Given: **SSS** (finding the angle)

Ex: Find the largest angle

Find $\angle A$.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$23^2 = 18^2 + 15^2 - 2(18)(15) \cos A$$

$$529 = 324 + 225 - 540 \cos A$$

$$540 \cos A = 324 + 225 - 529$$

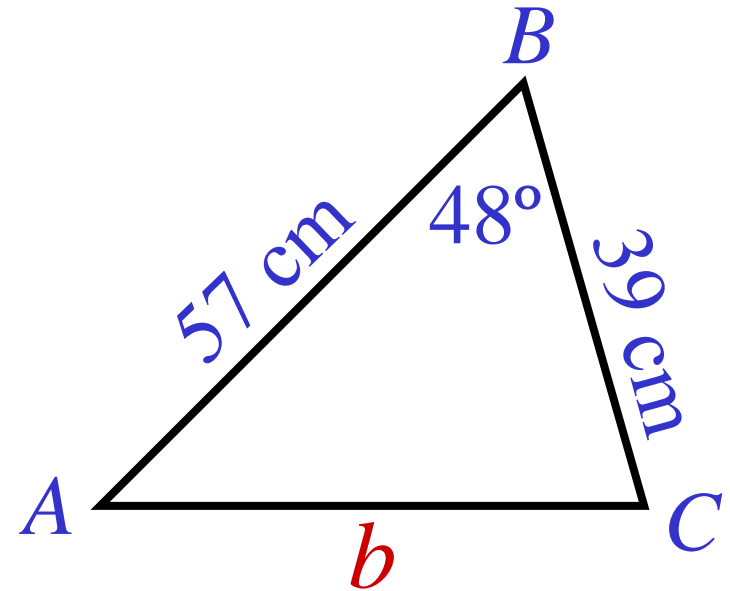
$$\cos A = \frac{324 + 225 - 529}{540}$$

$$\cos A = 0.0370$$

$$\angle A = \cos^{-1}(0.0370)$$

$$\angle A = 87.9^\circ$$

Ex: If $a = 39$ cm,
 $\angle B = 48^\circ$ and $c = 57$ cm,
Find b .



$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$b^2 = 39^2 + 57^2 - 2(39)(57) \cos 48^\circ$$

$$b^2 = 1521 + 3249 - 2975$$

$$b^2 = 1794$$

$$b = \sqrt{1794}$$

$$b = 42.4 \text{ cm}$$