6.3 The Sine Law

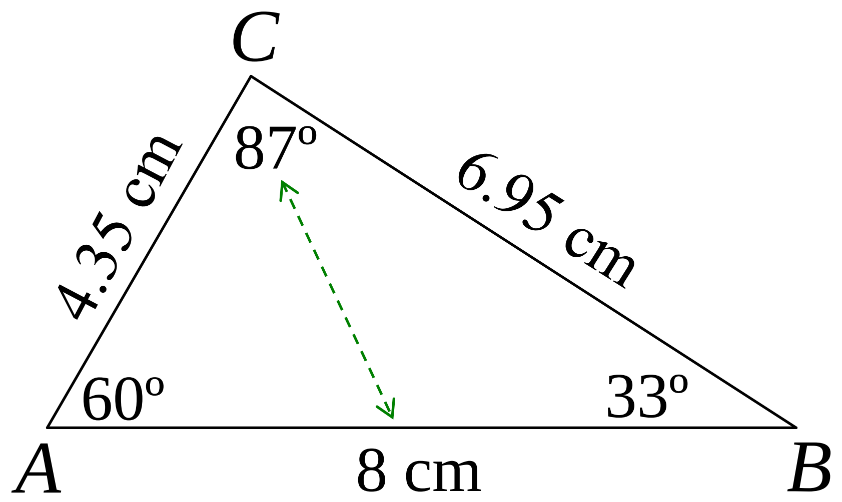
It is used to solve <u>*Oblique Triangles*</u>

(non-right triangles)

(the triangle <u>does not</u> contain a 90° angle)

It is used when you are given two angles and a side **(ASA)**, or two sides and an angle **(SSA)**.

Given: ΔABC.



The largest angle is opposite the longest side.

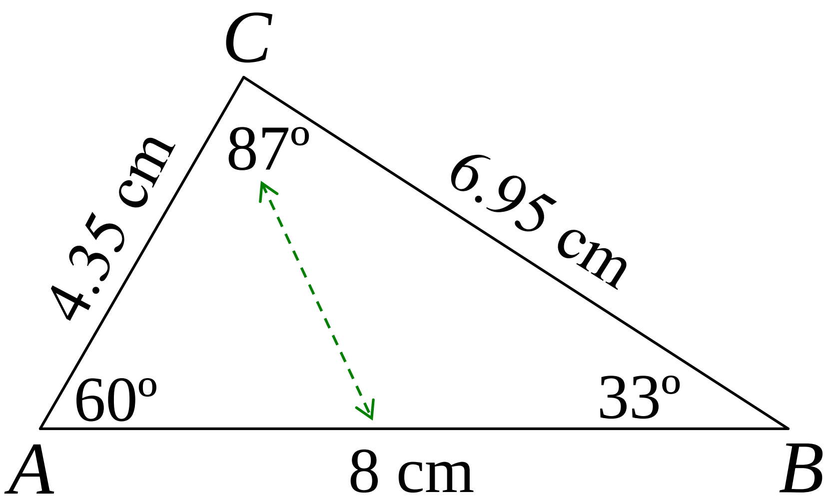
Are the ratios of the sine of the angles and opposite sides the same?

$$\frac{\sin 87^\circ}{8} = 0.125 \qquad \frac{\sin 60^\circ}{6.95} = 0.125 \qquad \frac{\sin 33^\circ}{4.35} = 0.125$$

Yes, they *are* in the same ratio.

Given: ΔABC.

The largest angle is opposite the longest side.

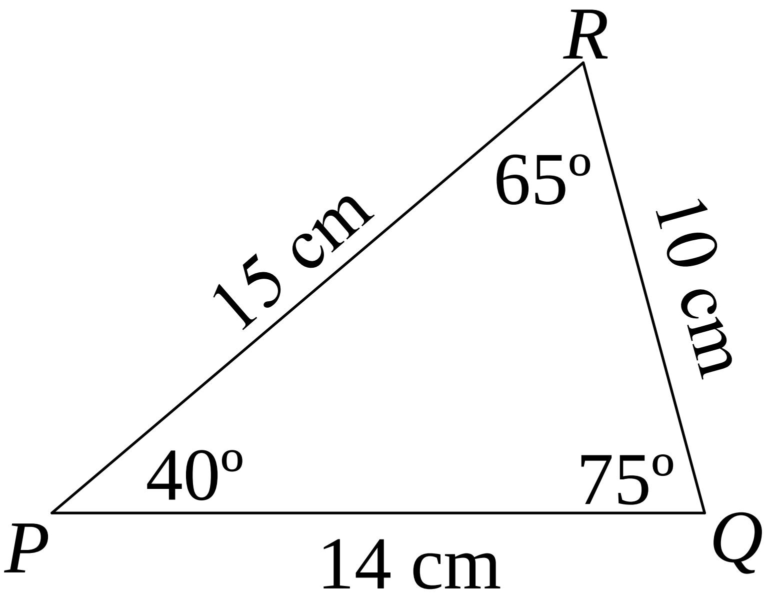


Note: the ratios of the **angles** (not the sine of the angles and opposite sides are *not* the same?

$$\frac{87^\circ}{8\,cm} = 10.9 \qquad \frac{60^\circ}{6.95\,cm} = 8.6 \qquad \frac{33^\circ}{4.35} = 7.6$$

Given: ΔPQR.

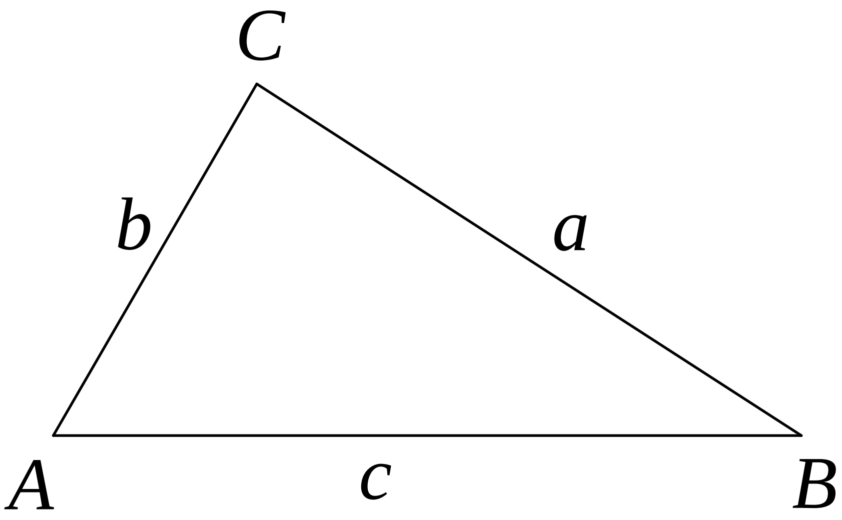
Determine the ratios of the sine of the angles and the opposite sides?



$$\frac{\sin 75^\circ}{15} = 0.06 \qquad \frac{\sin 65^\circ}{14} = 0.06 \qquad \frac{\sin 40^\circ}{10} = 0.06$$

They *are* in the same ratio.

The Sine Law

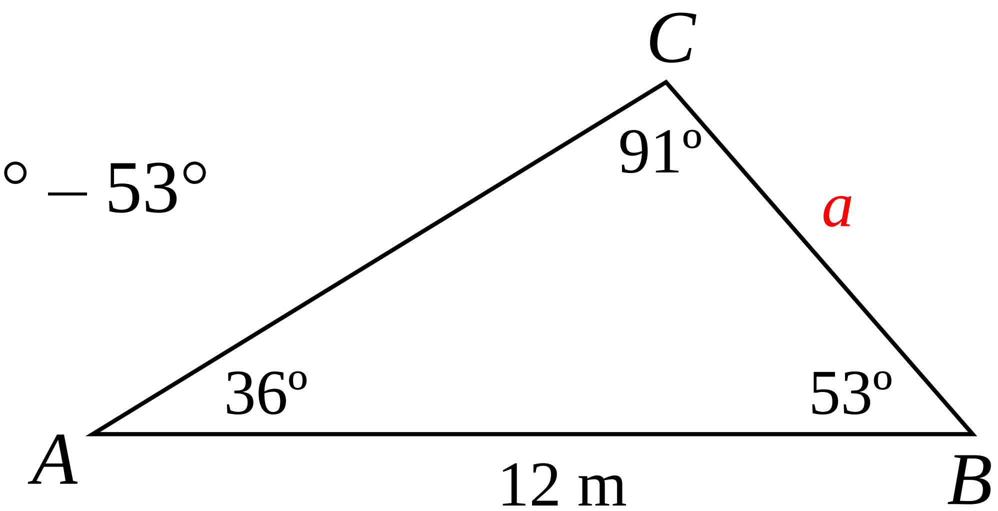


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad (1)$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad (2)$$

Example 1: Determine the length of side *a*.

Step 1: Determine $\angle C$.



$$\angle C = 180° - 36° - 53°$$

$$\angle C = 91°$$

Step 2: Set up the sine ratio.

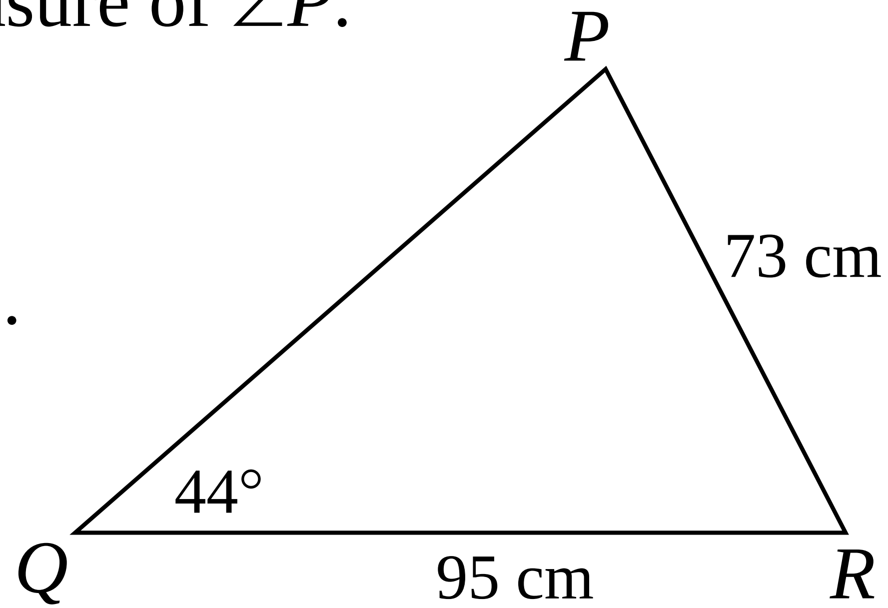
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 36°} = \frac{12}{\sin 91°}$$

$$a = \frac{12 \sin 36°}{\sin 91°}$$

$$a = 7 \text{ m}$$

Example 2: Given ΔPQR with $\angle Q = 44°$, $q = 73$ cm and $p = 95$ cm, find the measure of $\angle P$.



Step 1: Draw a sketch.

Step 2: Set up the sine ratio.

$$\frac{\sin P}{p} = \frac{\sin Q}{q}$$

$$\frac{\sin P}{95} = \frac{\sin 44^\circ}{73}$$

$$\sin P = \frac{95 \sin 44^\circ}{73}$$

$$\sin P = 0.9040$$

$$\angle P = \sin^{-1}(0.9040)$$

$$\angle P = 64.7°$$

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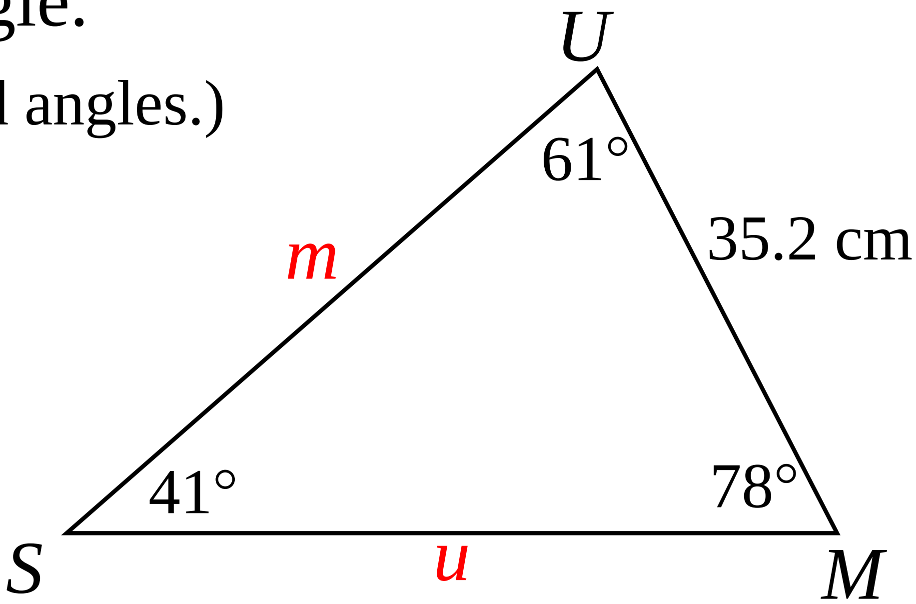
Example 3: Given ΔSUM with $\angle S = 41°$, $\angle M = 78°$, $s = 35.2$ cm. Solve the triangle.

(Find all the unknown sides and angles.)



Step 1: Find $\angle U$

$$\angle U = 180° - 41° - 78°$$

$$\boxed{\angle U = 61°}$$

$$\frac{m}{\sin 78°} = \frac{35.2}{\sin 41°}$$

$$m = \frac{35.2 \sin 78°}{\sin 41°}$$

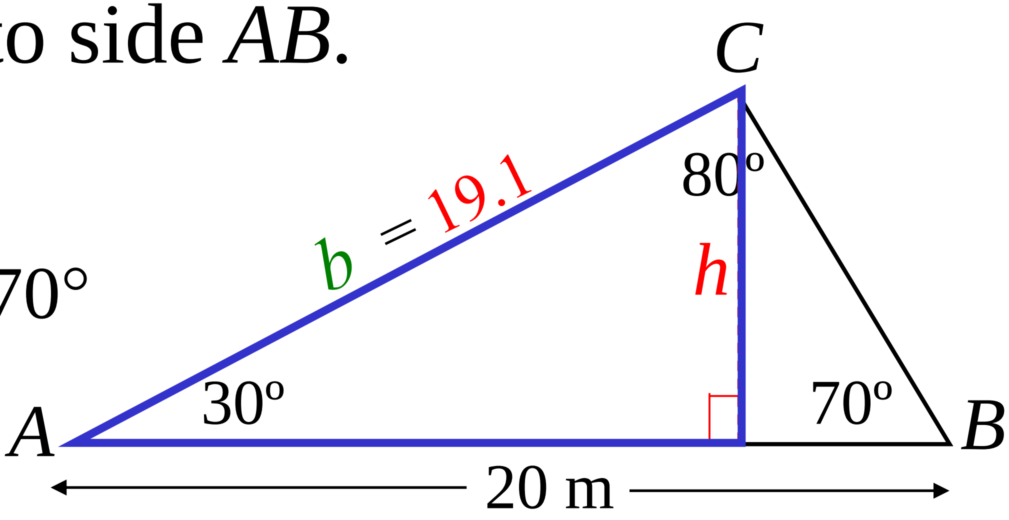
$$\boxed{m = 52.5 \text{ cm}}$$

$$\frac{u}{\sin 61°} = \frac{35.2}{\sin 41°}$$

$$u = \frac{35.2 \sin 61°}{\sin 41°}$$

$$\boxed{u = 46.9 \text{ cm}}$$

Example 4: Determine the length of the altitude from point C to side AB.



Step 1: Determine $\angle C$.

$$\angle C = 180° - 30° - 70°$$

$$\angle C = 80°$$

Step 2: Find side AC (b).

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 70^\circ} = \frac{20}{\sin 80^\circ}$$

$$b = \frac{20 \sin 70^\circ}{\sin 80^\circ}$$

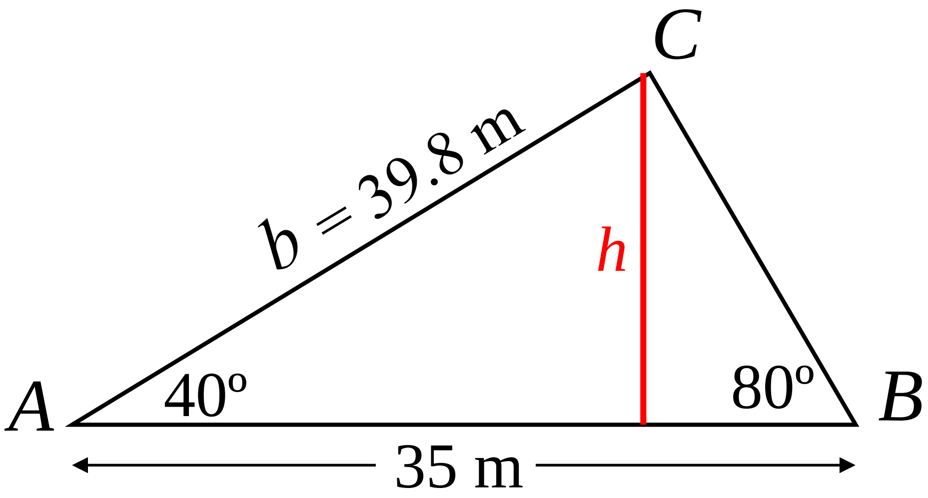
$$b = 19.1 \text{ m}$$

$$\sin 30^\circ = \frac{h}{19.1}$$

$$h = 19.1 \sin 30^\circ$$

$$\boxed{h = 9.5\,\text{m}}$$

Example 5: Determine the area of ΔABC.



Step 1: Determine $\angle C$.

$$\angle C = 180^\circ - 40^\circ - 80^\circ$$

$$\angle C = 60^\circ$$

Step 2: Find side AC (b).

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 80^\circ} = \frac{35}{\sin 60^\circ}$$

$$b = \frac{35 \sin 80^\circ}{\sin 60^\circ}$$

$$b = 39.8 \text{ m}$$

$$\sin 40^\circ = \frac{h}{39.8}$$

$$h = 39.8 \sin 40^\circ$$

$$h = 25.6 \text{ m}$$

$$A = \frac{bh}{2} = \frac{35 \times 25.6}{2}$$

$$A = 448 \text{ m}^2$$