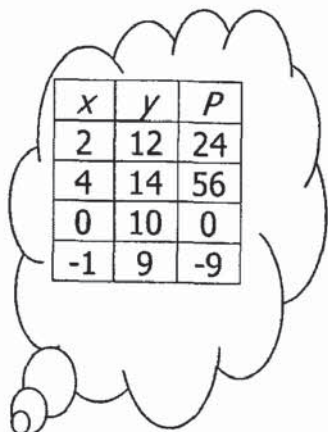


Example:

Two numbers differ by 10. What would the numbers be to produce the least possible product?



x	y	P
2	12	24
4	14	56
0	10	0
-1	9	-9

Step 1: Identify the quantity that you want to minimize or maximize.

Here you want to find the minimum product "**P**", of two numbers.

Step 2: Write the algebraic expression for this quantity.

Let x and y represent the two numbers

$$P = xy$$

Step 3: From the given information, eliminate one of the variables

Since the numbers differ by 10, $y = x - 10$

$$P = x(x - 10)$$

Step 4: The expression in step 3 is a quadratic function. Rearrange and complete the square.

$$P = x(x - 10)$$

$$= x^2 - 10x$$

$$P + 25 = x^2 - 10x + 25$$

$$P = (x - 5)^2 - 25$$

Step 5: State the maximum or minimum value of the function and the value of x for which it occurs.

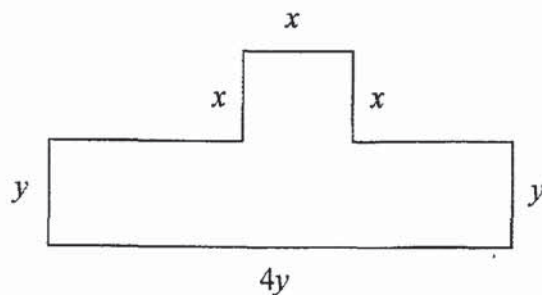
Since $a > 0$, the **minimum** value is -25 , when $x = 5$.

Step 6: Answer the question.

Since $y = x - 10$, $y = -5$.

\therefore the minimum product is -25 when the numbers are 5 and -5 .

- The general equation of a thrown object is given by $h = h_0 + v_0t - 5t^2$, where the values of h_0 and v_0 represent the initial height and initial speed of the object.
 - Determine the equation representing the height of a rock that is thrown upward from a cliff that is 15 m high, at an initial speed of 10 m/s.
 - Determine the maximum height of the rock.
- A ball is thrown from an apartment building. Its height, in metres, after t seconds, is given by $h = -5t^2 + 10t + 35$.
 - Determine the initial height of the ball.
 - Determine the maximum height of the ball.
 - Determine the length of time it takes for the ball to reach that height.
- Two numbers differ by 8. Their product is to have the least value possible. Determine the numbers.
- The sum of the base and the height of a triangle is 15 cm. What is the greatest possible area for a triangle having this property.
- A rectangular lot is bordered on one side by a stream and on the other three sides by fencing. If there is 600 metres of fence available, determine the dimensions of the lot with the greatest area.
- A rectangular field is enclosed by a fence and divided into two lots by another section of fence parallel to two of its sides. If the 600 metres of fence that is used must enclose a maximum area, what are the dimensions of the field?
- A fence is to be built around the area shown in the diagram. Determine the values of x and y that would produce a minimum area if the perimeter is 300 metres.



Answers

- $h = -5t^2 + 10t + 15$; max height is 20 m
- 35 m; 40 m; 1 s
- 4, -4
- $\frac{225}{8}$ or 28.125
- 150 by 300 m; 45 000 m²
- 100 by 150 m
- $(\frac{750}{29}, \frac{600}{29})$

Maximize Revenue and Profit

- If the total costs are $C(x) = 500 + 90x$, and total revenues are $R(x) = 150x - x^2$, where x represents the total number of merchandise sold.
 - Find the break-even point(s).
 - Write the profit function, and find what level production maximizes the profit?
 - What is the maximum profit?
- Assume that a company knows that the cost to produce x items is given by the cost function $C(x) = 5x^2 + 800x$ dollars. It also knows that the revenue from x items is given by the revenue function $R(x) = 1000x + 200$. Find the maximum profit they can expect and how many of these items they have to produce and sell to make this maximum profit.
- For producing a certain product, if total costs can be represented by $C(x) = 1600 + 1500x$, and the total revenues can be represented by $R(x) = 1600x - x^2$, find the break-even point(s) and the maximum possible profit.
- An auditorium has seats for 1200 people. For the past several days, the auditorium has been filled to capacity for each show. Tickets currently cost \$5.00 and the owner wants to increase the ticket prices. He estimates that for each \$0.50 increase in price, 100 fewer people will attend. What ticket price will maximize the profit?
- A grocer sells 50 loaves of bread a day. The cost is \$0.65 a loaf. The grocer estimates that for each \$0.05 price increase, 2 fewer loaves of bread will be sold. Graph, and then determine what cost will maximize the profit?
- A bus company transports 500 people a day between Morse Rd. and high St. The one-way fare is \$0.50. The owner estimates that for each \$0.10 price increase, 50 passengers will be lost. Graph, and then determine what price will maximize their profit?
- The city transit system carries 24 800 bus riders per day for a fare of \$1.85. The city hopes to reduce car pollution by getting more people to ride the bus, while maximizing the transit system's revenue at the same time. A survey indicates that the number of riders will increase by 800 for every \$0.05 decrease in the fare. What fare will produce the greatest revenue?
- A senior's dance club has a \$5 cover charge and averages 300 customers on Friday nights. Over the past several months, the club has changed the cover price several times to see how it affects the number of customers. They have discovered that for every increase of \$0.50 in the cover charge, the number of customers decreases by 30, find the cover charge that maximize the revenue.
- The cost C , in dollars, of operating a concrete-cutting machine is modelled by $C = 2.2n^2 - 66n + 655$, where n is the number of minutes the machine is run. How long must the machine run for the operating cost to be at a minimum? What is the minimum cost?
- A bus company has 4000 passengers daily, each paying a fare of \$2. For each \$0.15 increase, the company estimates that it will lose 40 passengers. If the company needs to take in \$10 450 per day to stay in business, what fare should be charged?

Answers

- (50, 5000), (10, 1400)
 - producing 30 units maximizes the profit.
- The maximum profit is \$2,200
- The break-even points are (20, 31600), (80, 121600)
- Single ticket price = \$5.50; Maximum income = \$6050
- Single loaf price = \$0.95; Maximum income = \$36.10
- Single ticket price = \$0.75; Maximum income = \$281.25
- \$1.70
- keep the price, no change
- at 15 min, the minimum cost is \$160
- \$54.44 or \$3.33 per fare