4.6 Completing the Square

Three forms of a quadratic equation.

1) Factored Form

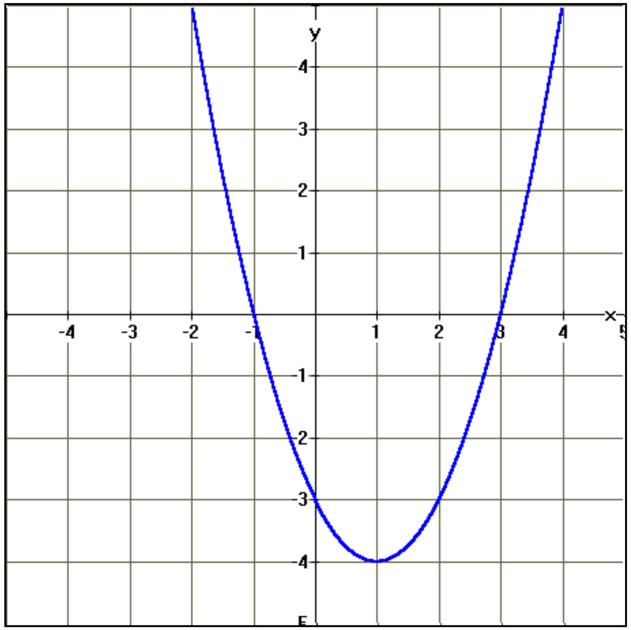
$y = (x + 1)(x - 3)$

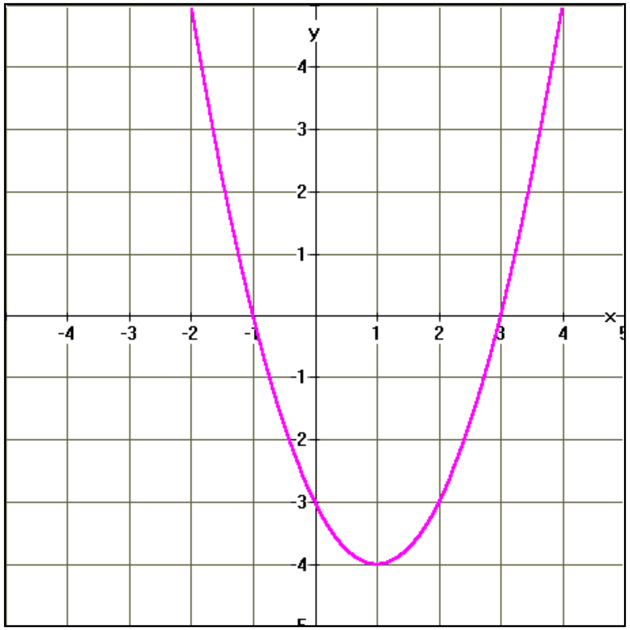
2) Vertex form

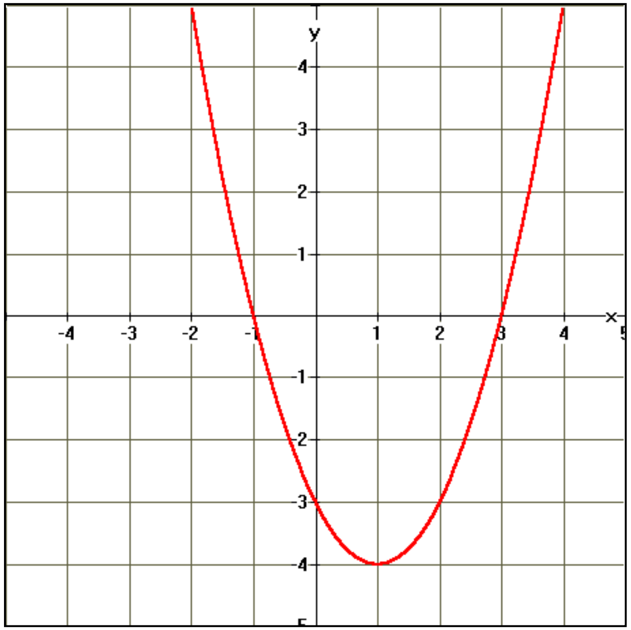
$y = (x - 1)^2 - 4$

3) Standard form

$y = x^2 - 2x - 3$







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Examples of *'perfect squares'*.

1. $(x + 3)^2 = x^2 + 6x + 9$

2. $(x - 4)^2 = x^2 - 8x + 16$

3. $(x + 5)^2 = x^2 + 10x + 25$

4. $(x - 6)^2 = x^2 - 12x + 36$

5. $(x + 7)^2 = x^2 + 14x + 49$

What term completes the square?

1. $x^2 + 6x +$? 9

2. $x^2 – 8x +$? 16

3. $x^2 + 10x +$? 25

4. $x^2 – 20x +$? 100

What term completes the square?

5. $x^2 + 22x +$? 121 $\qquad \left(\dfrac{22}{2}\right)^2$

6. $x^2 - x +$? $\dfrac{1}{4}$ $\qquad \left(\dfrac{1}{2}\right)^2$

7. $x^2 + 3x +$? $\dfrac{9}{4}$ $\qquad \left(\dfrac{3}{2}\right)^2$

8. $x^2 - 5x +$? $\dfrac{25}{4}$ $\qquad \left(\dfrac{5}{2}\right)^2$

Complete the square:

$y = x^2 - 14x$

complete the square

$$\left(\frac{14}{2}\right)^2$$

$y = x^2 - 14x + 49 - 49$

Factor the first three terms

$y = (x^2 - 14x + 49) - 49$

$y = (x - 7)^2 - 49$

Complete the square:

$y = 2x^2 + 12x + 5$ factor "*a*"

$y = 2(x^2 + 6x) + 5$ complete the square

$y = 2(x^2 + 6x + 9 – 9) + 5$ remove the 4th term

$y = 2(x^2 + 6x + 9) – 18 + 5$ vertex form

$y = 2(x + 3)^2 – 13$

The vertex is (– 3, – 13)

Complete the square:

$$y = \frac{1}{2}x^2 + 3x - 5$$

factor "*a*"

$$y = \frac{1}{2}(x^2 + 6x) - 5$$

complete the square

$$y = \frac{1}{2}(x^2 + 6x + 9 - 9) - 5$$

remove the 4th term

$$y = \frac{1}{2}(x^2 + 6x + 9) - 4.5 - 5$$

vertex form

$$y = \frac{1}{2}(x + 3)^2 - 9.5$$

The vertex is (– 3, – 9.5)

Example: Put $y = -2x^2 + 20x - 35$ in vertex form.

$y = -2x^2 + 20x - 35$

factor coefficient of x^2

$y = -2(x^2 - 10x) - 35$

add and subtract half the square of the x term

$y = -2(x^2 - 10x + 25 - 25) - 35$

remove the fourth term

$y = -2(x^2 - 10x + 25) + 50 - 35$

factor the perfect square

$y = -2(x - 5)^2 + 15$

The vertex is (5, 15)

Complete the square:

$y = 3x^2 + 12x + 1$ factor "*a*"

$y = 3(x^2 + 4x) + 1$ complete the square

$y = 3(x^2 + 4x + 4 - 4) + 1$ remove the 4th term

$y = 3(x^2 + 4x + 4) - 12 + 1$ vertex form

$y = 3(x + 2)^2 - 11$

The vertex is (– 2, – 11)

Complete the square:

$$y = \frac{1}{2}x^2 + 5x - 1$$

factor "*a*"

$$y = \frac{1}{2}(x^2 + 10x) - 1$$

complete the square

$$y = \frac{1}{2}(x^2 + 10x + 25 - 25) - 1$$

remove the 4th term

$$y = \frac{1}{2}(x^2 + 10x + 25) - 12.5 - 1$$

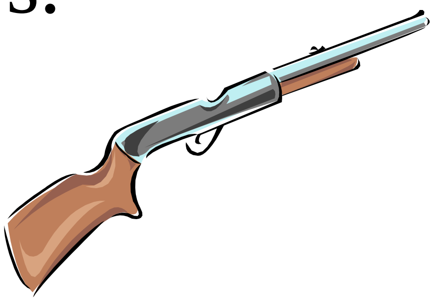
vertex form

$$y = \frac{1}{2}(x + 5)^2 - 13.5$$

The vertex is (– 5, – 13.5)

Problem

A bullet is fired vertically into the air at an initial velocity of 100 m/s. The height in metres is given by the $h = 100t - 5t^2$. (t = sec.) Find the maximum height attained by the bullet.



$$h = -5t^2 + 100t$$

$$h = -5(t^2 - 20t)$$

$$h = -5(t^2 - 20t + 100 - 100)$$

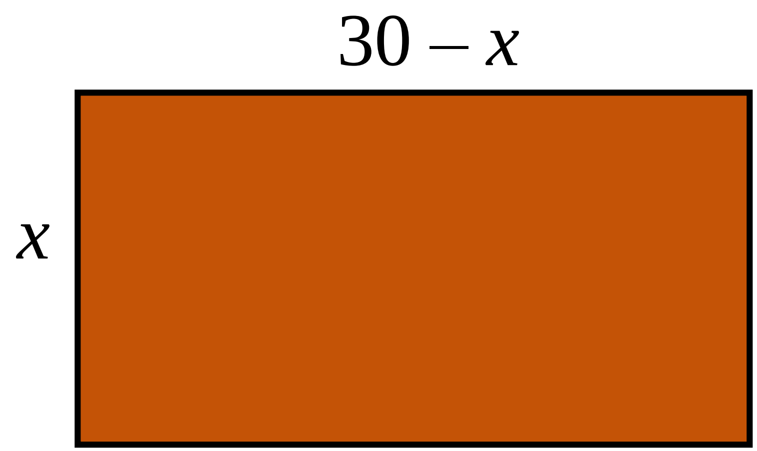
$$h = -5(t^2 - 20t + 100) + 500$$

$$h = -5(t - 10)^2 + 500$$

Vertex is (10, 500)

After 10 seconds the bullet reached a height of 500 m.

A rectangular lot has a perimeter of 60 m. Find the dimensions that will produce a maximum area.



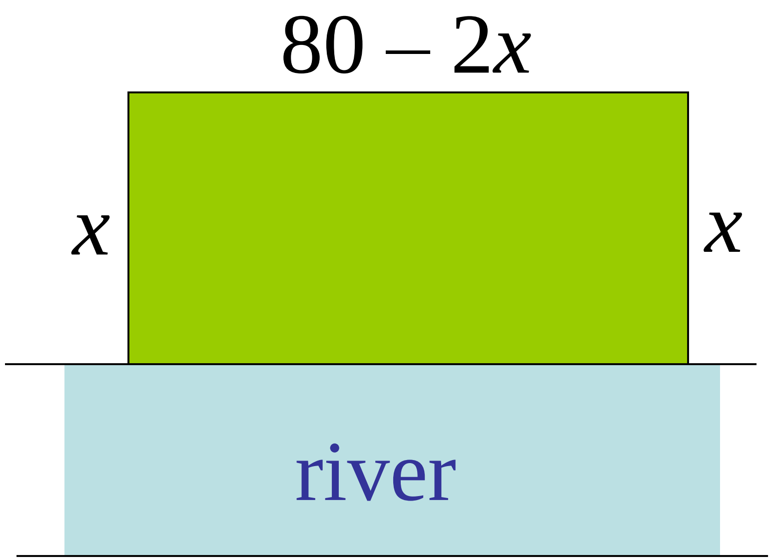
Area = length × width

$$\text{Area} = x(30 - x)$$
$$= 30x - x^2$$
$$= -x^2 + 30x$$
$$= -(x^2 - 30x)$$
$$= -(x^2 - 30x + 225 - 225)$$
$$= -(x^2 - 30x + 225) + 225)$$
$$= -(x - 15)^2 + 225$$

Vertex is (15, 225)

When the width is 15 m the area is 225 m^2

A farmer has 80 m of fencing to enclose a rectangular pen for his animals with a river on one side. What dimensions will produce a maximum area?



$$\text{Area} = x(80 - 2x)$$

$$= 80x - 2x^2$$

$$= -2x^2 + 80x$$

$$= -2(x^2 - 40x)$$

$$= -2(x^2 - 40x + 400 - 400)$$

$$= -2(x^2 - 40x + 400) + 800$$

$$= -2(x^2 - 20) + 800$$

$$x = 20$$

$$80 - 2x = 40$$

The dimensions are 20 m by 40 m.