

# Cube Factoring

Date: \_\_\_\_\_

## The Sum and Difference of Cubes

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

### Proof:

$$(x + y)^3 = (x + y)(x + y)^2$$

$$(x + y)^3 = (x + y)(x^2 + 2xy + y^2)$$

$$(x + y)^3 = x^3 + 2x^2y + xy^2 + yx^2 + 2xy^2 + y^3$$

$$(x + y)^3 = 3x^2y + 3xy^2 + x^3 + y^3$$

$$(x + y)^3 - 3x^2y - 3xy^2 = x^3 + y^3$$

$$(x + y)(x + y)^2 - 3xy(x + y) = x^3 + y^3$$

$$(x + y)[(x + y)^2 - 3xy] = x^3 + y^3$$

$$(x + y)(x^2 + 2xy + y^2 - 3xy) = x^3 + y^3$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

use similar steps

$$(x - y)^3 =$$

### Example 1

Factor

a)  $x^3 - 8 = x^3 - 2^3$

$$= (x - 2)(x^2 + 2x + 4)$$

this term  
will not be  
further factorable.

b)  $27y^3 + 64 = (3y)^3 + 4^3$

c)  $5u^3 - 40(x + y)^3$

$$= 5(u^3 - 8(x + y)^3)$$

$$= 5\{u^3 - [2(x + y)]^3\}$$

$$= 5\{[u - 2(x + y)][u^2 + 2u(x + y) + 4(x + y)^2]\}$$

$$= 5[(u - 2x - 2y)(u^2 + 2ux + 2uy + 4(x^2 + 2xy + y^2))]$$

$$= 5(u - 2x - 2y)(u^2 + 2ux + 2uy + 4x^2 + 8xy + 4y^2)$$

d)  $x^9 - y^9$

$$\begin{aligned} &\rightarrow (x^3)^3 - (y^3)^3 = \\ &= (x^3 - y^3)[(x^3)^2 + x^3y^3 + (y^3)^2] \\ &= (x^3 - y^3)(x^6 + x^3y^3 + y^6) \\ &= (x - y)(x^2 + xy + y^2)(x^6 + x^3y^3 + y^6) \end{aligned}$$

## Cube Factoring

Date: \_\_\_\_\_

## 1. Factor

a)  $a^3 - b^3$

b)  $a^3 + b^3$

c)  $c^3 + d^3$

d)  $x^3 - 8$

e)  $y^3 + 125$

f)  $z^3 - 1$

g)  $x^3 - a^3$

h)  $-x^3 + 1$

i)  $x^3 - 343$

j)  $27 - y^3$

k)  $8x^3 - 1$

l)  $a^3b^3 - 1$

m)  $y^3 - 125$

n)  $64 - 27a^3$

o)  $27m^3 + 1$

p)  $64 + 27b^3$

## 2. Factor fully.

a)  $5x^3 - 40$

b)  $2x^3 + 16y^3$

c)  $mc^3 + 1000m$

d)  $x^4 + x$

e)  $3y - 24y^4$

f)  $3a^4 - 81a$

g)  $y^6 + 8y^3$

h)  $54 - 2x^3$

i)  $x^3 - \frac{1}{8}$

## 3. Factor fully.

a)  $(a+b)^3 - a^3$

b)  $(x-1)^3 + 1$

c)  $(x+3)^3 - 8$

d)  $(y+1)^3 + 1000$

e)  $(x+2)^3 - (x-2)^3$

f)  $(x+1)^3 - (y-2)^3$

g)  $(x-1)^3 + (x+1)^3$

h)  $(a+b)^3 - (a-b)^3$

i)  $(a+2b)^3 - (a-2b)^3$

j)  $(3x-2y)^3 + (2x-3y)^3$

k)  $x^6 - y^6$

l)  $x^9 + y^9$

**ANSWERS**

1.  $(a-b)(a^2 + ab + b^2)$ ,  $(a+b)(a^2 - ab + b^2)$ ,  $(c+d)(c^2 - cd + d^2)$ ,  $(x-2)(x^2 + 2x + 4)$ ,  
 $(y+5)(y^2 - 5y + 25)$ ,  $(z-1)(z^2 + z + 1)$ ,  $(x-a)(x^2 + ax + a^2)$ ,  $(x+1)(x^2 - x + 1)$ ,  
 $(x-7)(x^2 + 7x + 49)$ ,  $(3-y)(9 + 3y + y^2)$ ,  $(2x-1)(4x^2 + 2x + 1)$ ,  $(ab-1)(a^2b^2 + ab + 1)$ ,  
 $(y-5)(y^2 + 5y + 25)$ ,  $(4-3a)(16 + 12a + 9a^2)$ ,  $(3m+1)(9m^2 - 3m + 1)$ ,  $(4+3b)(16 - 12b + 9b^2)$
2.  $5(x-2)(x^2 + 2x + 4)$ ,  $2(x+2y)(x^2 - 2xy + 4y^2)$ ,  $m(c+10)(c^2 - 10c + 100)$ ,  
 $x(x+1)(x^2 - x + 1)$ ,  $3y(1-2y)(1 + 2y + 4y^2)$ ,  $3a(a-3)(a^2 + 3a + 9)$ ,  $y^3(y+2)(y^2 - 2y + 4)$ ,  
 $2(3-x)(9 + 3x + x^2)$ ,  $\left(x - \frac{1}{2}\right)\left(x^2 + \frac{1}{2}x + \frac{1}{4}\right)$
3.  $b(3a^2 + 3ab + b^2)$ ,  $x(x^2 - 3x + 3)$ ,  $(x+1)(x^2 + 8x + 19)$ ,  $(y+11)(y^2 - 8y + 91)$ ,  
 $4(3x^2 + 4)$ ,  $(x-y+3)(x^2 + xy + y^2 - 3y + 3)$ ,  $2x(x^2 + 3)$ ,  $2b(3a^2 + b^2)$ ,  
 $4b(3a^2 + 4b^2)$ ,  $5(x-y)(12x^2 - 11xy + 2y^2)$ ,  $(x-y)(x+y)(x^4 + x^2y^2 + y^4)$ ,  
 $(x+y)(x^2 - xy + y^2)(x^6 - x^3y^3 + y^6)$