

Common Factoring and Factoring by Grouping

Learning Target: We are learning to... factor using the techniques of common factoring, and grouping.

Success Criteria: I can... factor using the techniques of common factoring, and grouping.

Let's talk English. What do the words, "common" and "factor" mean?

"Common"

- similar
- share something the same

"Factor"

- write as a product
- multiples of other numbers

In other words, we are re-writing an expression in terms of its factors. This would be the OPPOSITE of expanding.

$$\begin{array}{c}
 \xrightarrow{\text{EXPANDING}} \\
 2x(x + 4y - 5) = 2x^2 + 8xy - 10x \\
 \xleftarrow{\text{FACTORING}}
 \end{array}$$

Common factoring should always be the first type of factoring you do (yes, there are more types). A polynomial is completely factored when no more variable factors can be removed and no more integer factors other than 1 or -1 can be removed.

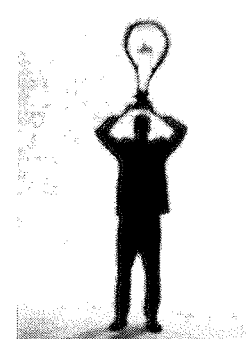
Let's take a look at some examples.

Ex. 1 Find the greatest common factor (GCF) of $3x^5$, $9x^4y^6$, $12x^2y$.

Greatest common factor of coefficients: 3

Greatest common factor of variable parts: x^2

Therefore, the greatest common factor of the polynomials is: $3x^2$



Ex. 2 Monomial Common Factor

Factor fully.

a) $2x + 6x^2$

$= 2x(1 + 3x)$

↑
GCF

b) $6x^2 + 12x$

$= 6x(x + 2)$

↑
GCF

c) $8x^3 - 6x^2y^2 + 4x^2y$

$= 2x^2(4x - 3y^2 + 2y)$

↑
GCF

d) $-15x^2y^2 + 10x^3y^2 - 5xy$

$= -5xy(3xy - 2x^2 + 1)$

↑
GCF.

MPM2D: Common Factoring and Factoring by Grouping

Ex. 3: Binomial Common Factor

Factor fully.

$$\begin{aligned} \text{a) } & 3x(2y+1) + 4m(2y+1) \\ &= (2y+1)(3x+4m) \\ & \quad \uparrow \\ & \text{GCF} \end{aligned}$$

$$\begin{aligned} \text{b) } & 8y(3z-4) - (3z-4) \\ &= (3z-4)(8y-1) \\ & \quad \uparrow \\ & \text{GCF} \end{aligned}$$

$$\begin{aligned} \text{c) } & (3y-1)^2 + y(3y-1) \\ &= (3y-1)(3y-1+y) \\ &= (3y-1)(4y-1) \\ & \quad \uparrow \\ & \text{GCF} \end{aligned}$$

Ex. 4: Factoring by Grouping

Factor fully.

$$\begin{aligned} \text{a) } & ac + bc + ad + bd \\ &= (ac+bc) + (ad+bd) \\ &= c(a+b) + d(a+b) \\ &= (a+b)(c+d) \\ & \quad \uparrow \\ & \text{GCF} \end{aligned}$$

Another approach:

$$\begin{aligned} &= (ac+ad) + (bc+bd) \\ &= a(c+d) + b(c+d) \\ &= (c+d)(a+b) \\ & \quad \uparrow \\ & \text{GCF} \end{aligned}$$

$$\begin{aligned} \text{b) } & 2m^2 + 6n + 4m + 3mn \\ &= (2m^2+4m) + (6n+3mn) \\ &= 2m(m+2) + 3n(2+m) \\ &= (m+2)(2m+3n) \\ & \quad \uparrow \\ & \text{GCF} \rightarrow (m+2) = (2+m) \end{aligned}$$

$$\begin{aligned} \text{c) } & (x+1)^2 - x - 1 \\ &= \text{NOT FACTORABLE} \rightarrow \text{if } \begin{aligned} & (x+1)^2 - x - 1 \\ &= (x+1)^2 - (x+1) \\ &= (x+1)((x+1)-1) \\ &= (x+1)(x) \\ & \text{or } x(x+1) \end{aligned} \end{aligned}$$