2.11 Verifying Geometric Properties

When proving sides of a quadrilateral are parallel use **slope**

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

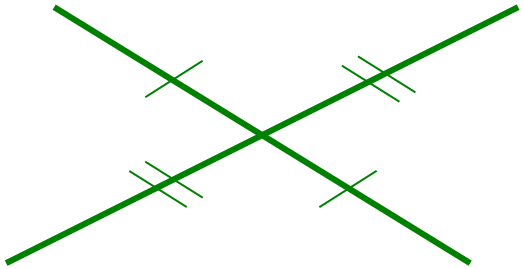
When proving sides are perpendicular, use **slope**

When proving sides are equal in length, use **distance**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Proving two line segments bisect each other, use midpoint

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



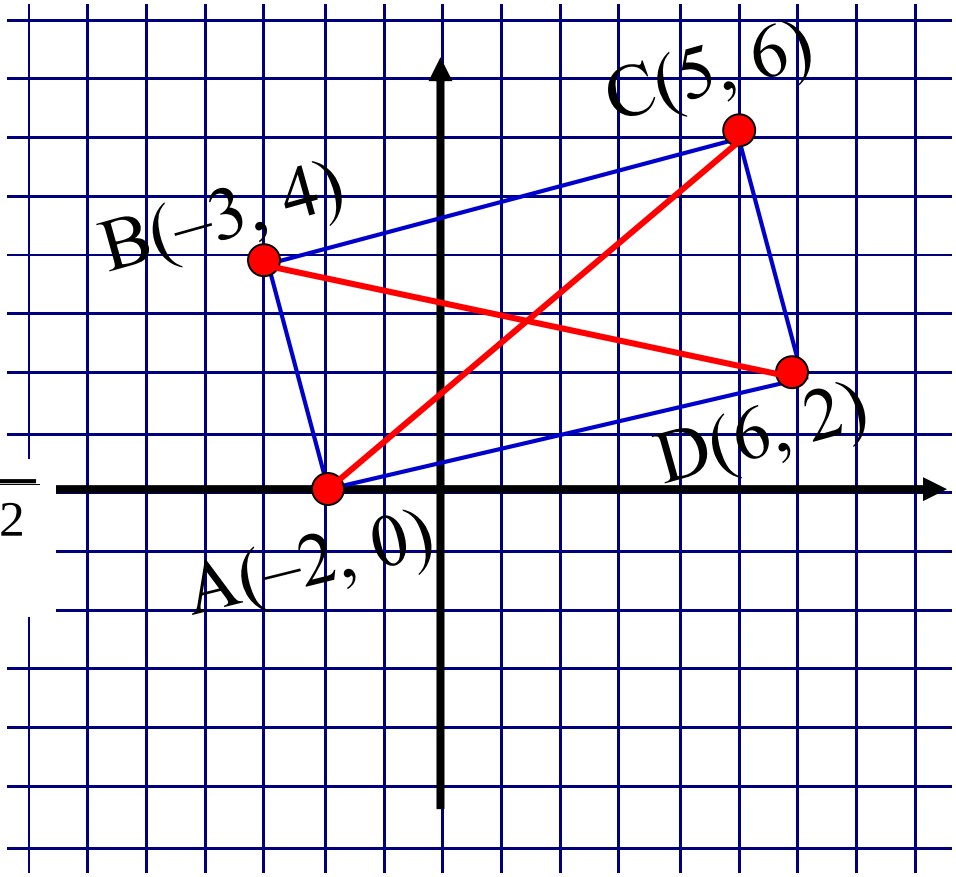
Ex. 1: Given rectangle A(–2, 0), B(–3, 4), C(5, 6), D(6, 2). Prove that the diagonals of rectangle ABCD are equal in length. Find the lengths of AC and BD

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_{AC} = \sqrt{(5+2)^2 + (6-0)^2}$$

$$d_{AC} = \sqrt{49 + 36}$$

$$\boxed{d_{AC} = \sqrt{85}}$$



$$d_{BD} = \sqrt{(-3-6)^2 + (4-2)^2}$$

$$d_{BD} = \sqrt{81 + 4}$$

$$\boxed{d_{BD} = \sqrt{85}}$$

Ex. 2: Given rectangle A(–2, 0), B(–3, 4), C(5, 6), D(6, 2).
Prove that the diagonals bisect each other.
Find the midpoints of AC and BD

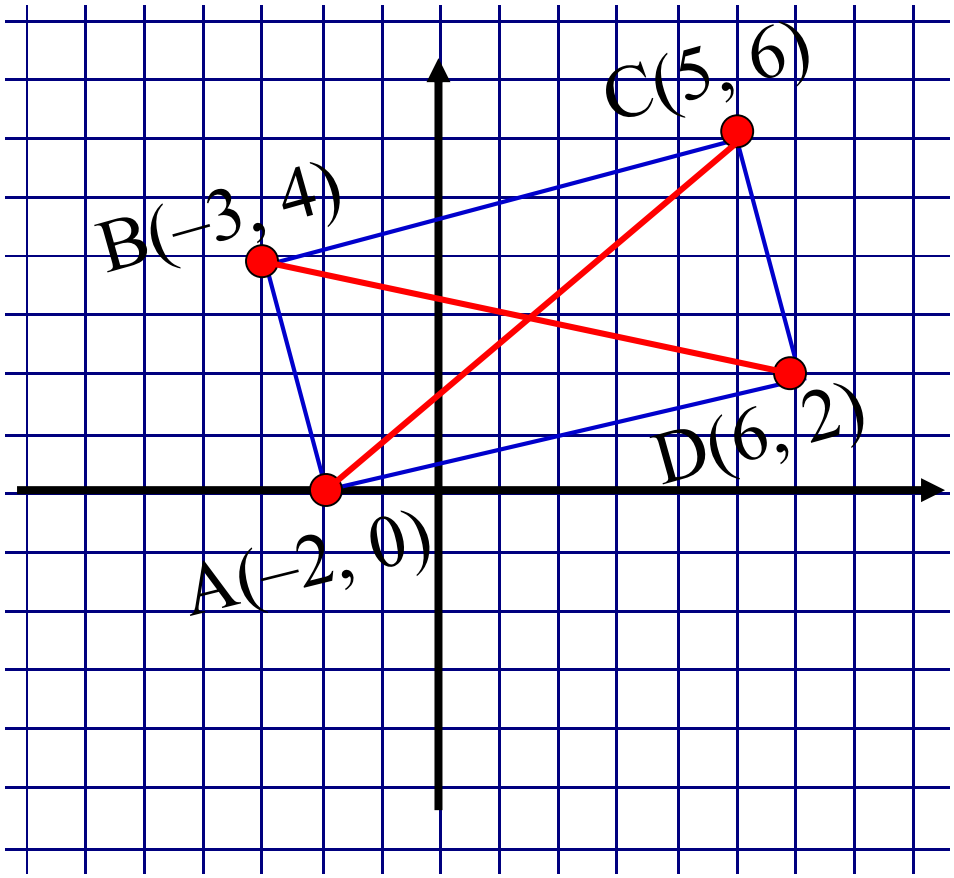
$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M_{AC}=\left(\frac{-2+5}{2},\frac{0+6}{2}\right)$$

$$M_{AC}=(1.5,3)$$

$$M_{BD}=\left(\frac{-3+6}{2},\frac{4+2}{2}\right)$$

$$M_{BD}=(1.5,3)$$



Ex. 3: Prove the midsegments of P (–4, 5), Q(6, 7), R(4,–3), S(–6,–1) form a parallelogram.

Find the midpoints of the sides

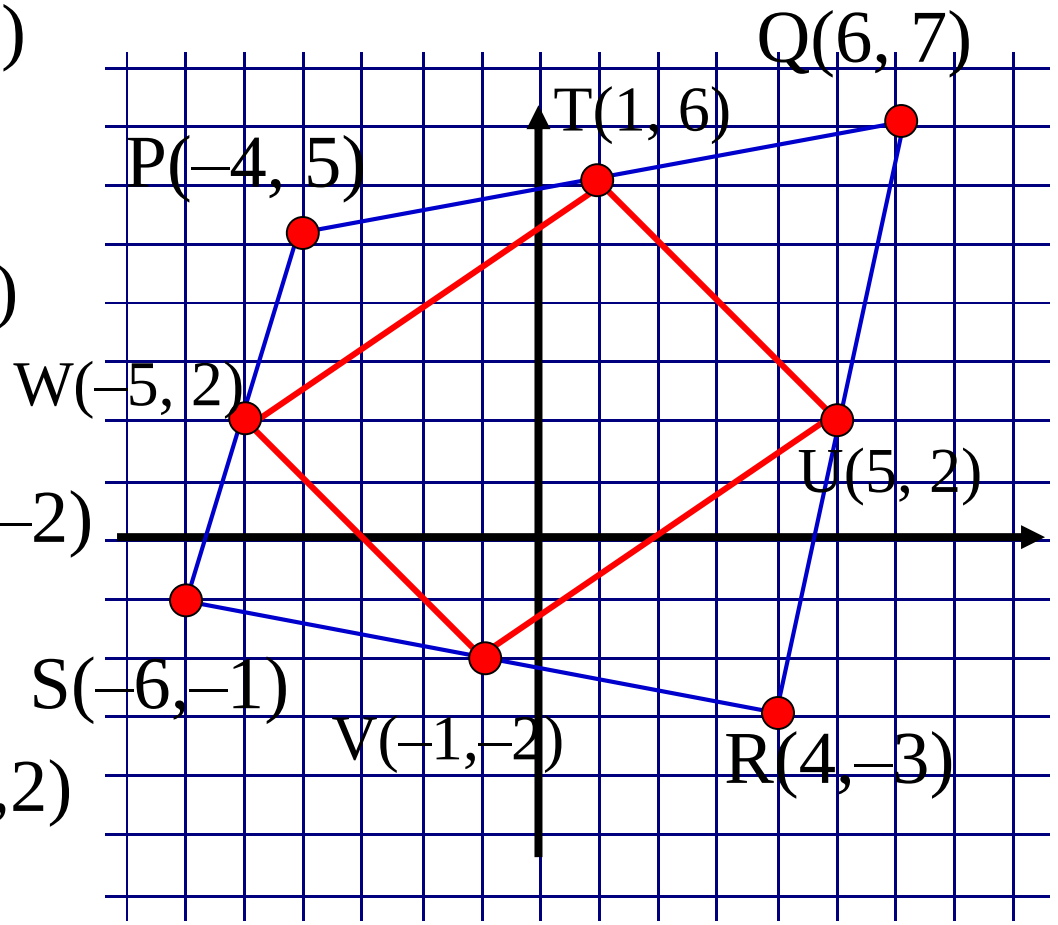
$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M_{PQ}=\left(\frac{-4+6}{2},\frac{5+7}{2}\right) \quad \text{T(1,6)}$$

$$M_{QR}=\left(\frac{6+4}{2},\frac{7-3}{2}\right) \quad \text{U(5,2)}$$

$$M_{RS}=\left(\frac{4-6}{2},\frac{-3-1}{2}\right) \quad \text{V(–1,–2)}$$

$$M_{SP}=\left(\frac{-6-4}{2},\frac{-1+5}{2}\right) \quad \text{W(–5,2)}$$



T(1,6) U(5,2) V(–1,–2) W(–5,2)

Find the slopes of *TUVW*

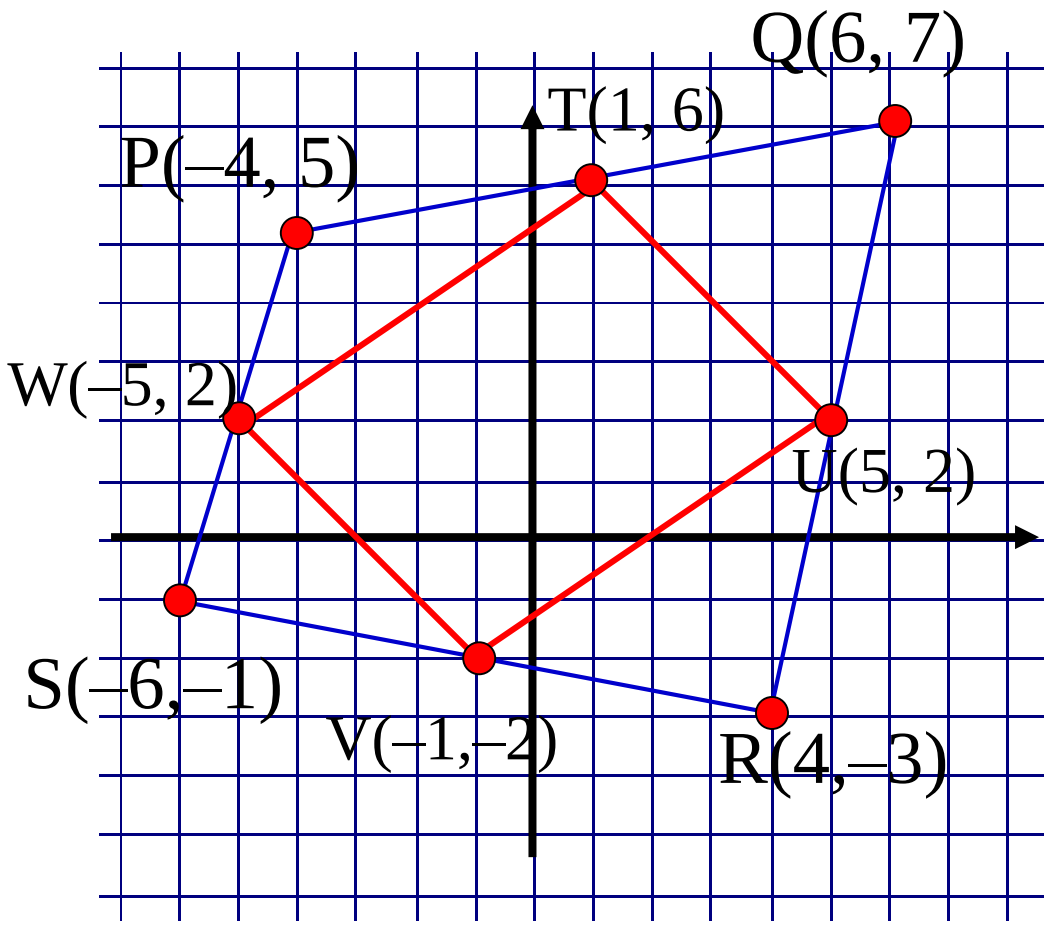
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{TU} = \frac{2-6}{5-1} = -1$$

$$m_{UV} = \frac{2+2}{5+1} = \frac{2}{3}$$

$$m_{VW} = \frac{2+2}{-5+1} = -1$$

$$m_{WT} = \frac{6-2}{1+5} = \frac{2}{3}$$



Ex. 4: Show that the points $A(1, 4)$ $B(3, 0)$ and $C(9, 8)$ lie on a circle with centre (6, 4).

Strategy: Find the circumcentre of triangle ABC and confirm the circumcentre is the point (6, 4).

I: Find the equation of the perpendicular bisector of AB.

midpoint of AB is (2, 2)

slope of AB is -2.

slope of perp. bisector is $\frac{1}{2}$

$$y = \frac{1}{2}x + b$$

$$2 = \frac{1}{2}(2) + b$$

$$1 = b$$

$$\boxed{y = \frac{1}{2}x + 1}$$

Ex. 4: Show that the points $A(1, 4)$ $B(3, 0)$ and $C(9, 8)$ lie on a circle with centre (6, 4).

II: Find the equation of the perpendicular bisector of AC.

midpoint of AC is (5, 6)

slope of AC is $\frac{1}{2}$.

slope of perp. bisector is -2

$$y = -2x + b$$

$$6 = -2(5) + b$$

$$16 = b$$

$$\boxed{y = -2x + 16}$$

Ex. 4: Show that the points $A(1, 4)$ $B(3, 0)$ and $C(9, 8)$ lie on a circle with centre (6, 4).

III: Solve by substitution.

$$y = \frac{1}{2}x + 1 \qquad y = -2x + 16$$

$$\frac{1}{2}x + 1 = -2x + 16$$

$$\times 2: \quad x + 2 = -4x + 32$$

$$x + 4x = 32 - 2$$

$$5x = 30$$

$$x = 6$$

$$y = -2(6) + 16$$

$$y = -12 + 16$$

$$y = 4$$

The circumcentre is (6, 4), which is what was given as the centre of the circle.