Present Value:

IF $F = P(1+i)^n$, then by rearranging for P,

$$P = \frac{F}{(1+i)^n} \quad \text{OR} \quad P = F(1+i)^{-n}$$

EX(4): To have $22000 in 5 yrs, how much money must be invested today at 5.1% /a Cs?

Soln: $P = \frac{F}{(1+i)^n}$

$$P = \frac{22000}{\left(1+\frac{0.051}{2}\right)^{5\times 2}}$$

$P = \$17\,102.75$

$\therefore$ \$17 102.75 must be invested today.

Present Value: In some situations, we need to know how much money to invest (OR borrow) today to produce a desired amount at a later date. This is called the present value "PV" of an amount

$$P = \frac{F}{(1+i)^n}$$

CP- P. ~~156~~ 92 EX's

1. $P = \frac{30000}{(1.08)^{15}}$

$P = \$9457.25$

2. $P = \frac{10000}{\left(1+\frac{0.06}{4}\right)^{4\times 1.75}}$

$= \$9010.27$

3. For the first 2 yrs

$P = \frac{8000}{\left(1+\frac{0.07}{12}\right)^{2\times 12}}$

$= \$6957.70$

For the next 3 yrs.

$P = \frac{6957.7}{\left(1+\frac{0.09}{2}\right)^{2\times 3}}$

$P = \$5342.78$

Present Value Examples Date:

∵ $F = P(1+i)^n$, then $P = \frac{F}{(1+i)^n} = F(1+i)^{-n}$

1. I want to have \$30 000 in 15 years from now. If I find an investment which will pay interest at 8%/a C_a, how much must I invest today?

Given: $F = \$30\,000$, $n = 15$, $i = 8\%$ compounded annually $\hookrightarrow i = 0.08$, $P = ?$

We could use $F = P(1+i)^n$ OR $P = \frac{F}{(1+i)^n}$ OR $P = F(1+i)^{-n}$

I will use the last one,

$P = F(1+i)^{-n}$

$= 30\,000(1+0.08)^{-15}$ ▷ $P = 30\,000(1.08)^{-15}$, you could enter the #'s into calculator in the same order that you see, using bracket.

$P = 9457.25$

∴ \$9457.25 must be invested today.

2. In 1¾ years, Rosie turns 21. At that time she will inherit \$10 000 from her grandmother. She does not want to wait until she is 21 to buy a car, instead she plans to borrow the money from a rich friend. She promises to give the rich friend all of the \$10 000 when she inherits it and agrees that the friend should earn interest on the money at 6%/a C_q. How much does the friend give Rosie today?

Given: $F = \$10000$, $i = 6\%$ compounded quarterly $\hookrightarrow i = \frac{0.06}{4} = 0.015$, $n = 1.75 \times 4 = 7$

$P = \frac{10000}{(1+0.015)^7}$

$P = \frac{10000}{(1.015)^7}$

$= 9010.27$

∴ Rosie's friend should give her \$9010.27 today.

3. Reinhold wants to have \$8 000 in 5 years from today. What single amount must he invest today if interest rates will be 7%/a C_m for the first 2 years and 9%/a C_s for the last 3 years?

For 1st 2 yrs:

$F = 8000$, $i = 7\%$ $c_m = \frac{0.07}{12}$, $n = 2 \times 12$, $n = 24$

$P = 8000\left(1 + \frac{0.07}{12}\right)^{-24}$

$= 8000(1.005833)^{-24}$

$P = 6957.70$

This amount becomes the future amount for the last 3 yrs.

For the last 3 yrs,

$F = 6957.70$, $i = \frac{0.09}{2} = 0.045$, $n = 3 \times 2 = 6$, $P = ?$

$P = \frac{6957.70}{(1+0.045)^6}$

$P = 5342.79$

∴ Reinhold must invest \$5342.79 today.