MCV4U Practice Exam Solutions

Exam Instructions:

1. **Answer all questions in the space provided (*none is provided on this practice exam, but there will be room on the "real exam"*).**
2. **If you need additional paper or graph paper ask for it, otherwise all answers should be written on this exam paper.**
3. **Only write answers on the front of any page and in dark pencil, black or blue pen.**
4. **You may use any handheld calculator (including the TI 83 Plus) for this entire exam.**
5. **The student may use on this exam a cheat sheet that is no larger than an 8.5 inch by 11 inch piece of paper, written on one side only.**
6. **10 additional communication marks are overall throughout the exam.**

MCV4U eLearning Final Exam Outline

The unit breakdown is as follows:

Unit	Marks
Unit 1 **Vectors**	15
Unit 2 **Equations of Lines and Planes**	15
Unit 3 **Rates of Change**	18
Unit 4 **Derivatives of Sinusoidal and Exponential Functions**	13
Unit 5 **Derivatives**	17
Unit 6 **Curve Sketching**	12
Overall Communication	10
Total Marks	100

There are 20 multiple choice questions in part 1 of the exam covering almost 2 and a half pages (letter size), and then part B has 9 and a half pages of 16 questions with room left to do all work.

Part I: Place your answers for Part I in the space provided.

1. Which of the following is a vector quantity? (B)
   A. time B. force C. mass D. temperature E. none of these

2. Using the ***Triangle Law of Addition*** to add two vectors, the vectors must be drawn (D)
   A. tail to head B. head to head C. tail to tail
   D. head to tail E. backwards

3. Given the points B(-3, 1, -2) and C(3, -5, 10) the vector $\overrightarrow{BC}$ in component form is (A)
A. [6, -6, 12] B. [-6, 6, -12] C. [-5, -4, 8] D. [1, -1, 2] E. [-1, 1, -2]

4. Given $|\vec{a}| = 5$, $|\vec{b}| = 6$ and the angle between them is 60°, then $\vec{a} \bullet \vec{b} =$ (A)
A. 15 B. 30 C. 51.96 D. 5.5 E. none of these

5. If $\vec{d} \times \vec{h} = [2, -6, 7]$, then $\vec{h} \times \vec{d} =$ (C)
A. $[2, -6, 7]$ B. $[7, -6, 2]$ C. $[-2, 6, -7]$ D. $[-6, 7, 2]$ E. none of these

6. Which of the following are parametric equations of a line through (3,-2) and (9,-4)? (D)
A. $x = 3 + 9t$, $y = -2 - 4t$ B. $x = 3 - 9t$, $y = -2 + 4t$ C. $x = -3 + 6t$, $y = +2 - 2t$ D. $x = 9 + 3t$, $y = -4 - t$ E. none of these

7. Which of the following points is on the plane $3x + y - 4z - 5 = 0$? (B)
A. (3, 1 –4) B. (2, -1, 0) C. (1, -3, -1) D. (-3, 7, 2) E. none of these

8. Which of the following yields a vector perpendicular to both original vectors? (C)
A. dot product B. scalar product C. cross product E. none of these

9. The derivative of the function $y = f(x)$ where $x = 3$ is (D)
A. $\lim_{\Delta x \to 0} \frac{f(x+a) - f(x)}{\Delta x}$ B. $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
C. $\lim_{h \to 0} \frac{f(x+3) - f(x)}{h}$ D. $\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$ E. none of these

10. The Power Differentiation Rule is (A)
A $\frac{d}{dx}(x^n) = nx^{n-1}$ B. $\frac{d}{dx}(k) = 0,\ k \in R$
C. $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$, if u is a function of x
D. $\frac{d}{dx}(u^n) = nu^{n-1}$ E. none of these

11. If $f(x) = r(x)h(x)$ then $f'(x) =$ (D)
A. $r(x)h'(x) - h(x)r'(x)$ B. $r'(x)h'(x)$

C. $h(x)r'(x)-r(x)h'(x)$ D. $r'(x)h(x)+h'(x)r(x)$ E. none of these

12. If $y=f(g(x))$, then $\frac{dy}{dx}=$ (E)

A. $f(g'(x))$ B. $f(g'(x))g(x)$ C. $f'(g(x))$

D. $f'(g'(x))$ E. none of these

13. The graph of the function $y=f(x)$ is always concave up where (D)

A. $f'(x)=0$ B. $f''(x)=0$ C. $f''(x)<0$

D. $f''(x)>0$ E. $f'(x)\le 0$

14. $\frac{d}{dx}\left[2e^{3x}\right]=$ (D)

A. $2e^{3x}$ B. e^{3x} C. $2e^{x}$ D. $6e^{3x}$ E. none of these

15. If $y=\sin 5x$, then $y'=$ (D)

A. $-\cos 5x$ B. $5\cos x$ C. $-5\cos 5x$ D. $5\cos 5x$ E. none of these

16. What kind of line shows an instantaneous rate of change the best? (B)

A. a secant line B. a tangent line

17. $\frac{d}{dx}\left[5^{4x^2}\right]=$ (B)

A. $8x5^{4x^2}$ B. $8x(\ln 5)5^{4x^2}$ C. $4x^2 5^{4x^2-1}$

D. $8x5^{4x^2-1}$ E. $(\ln 5)5^{8x}$

18. An interval upon which a function has a negative derivative it is said to be ______. (D)

A. concave up B. increasing C. concave down

D. decreasing E. none of these

19. An interval upon which a function has a positive second derivative the function is said to be __________. (A)

A. concave up B. increasing C. concave down

D. decreasing E. none of these

20. Which of the following is **not** a proper symbol for the derivative? (C)

A. y' B. $\frac{dy}{dx}$ C. $\frac{d}{dx}$ D. D_x E. $h'(x)$

Part II: Place your answers for Part II in the space provided on this exam paper. If you run out of room for any question, additional work can be shown on foolscap or lined paper.

1. Given the vectors $\vec{p} = [3,-1,4]$ and $\vec{u} = [-2,5,1]$. Find the angle between the vectors $\vec{p}$ and $\vec{u}$.

$$\vec{p} \bullet \vec{u} = 3(-2) - 1(5) + 4(1) = -7$$

$$|\vec{p}| = \sqrt{3^2 + (-1)^2 + 4^2} = \sqrt{26}$$

$$|\vec{u}| = \sqrt{(-2)^2 + 5^2 + 1^2} = \sqrt{30}$$

$$\cos\theta = \frac{\vec{p} \bullet \vec{u}}{|\vec{p}||\vec{u}|}$$

$$\cos\theta = \frac{-7}{\sqrt{26}\sqrt{30}}$$

$$\cos\theta = \frac{-7}{\sqrt{780}}$$

$$\cos\theta \doteq -0.2506$$

$$\therefore \theta \doteq 104.5^\circ$$

The angle between the vectors is about 104.5º.

2. Monster truck Alpha pulls on an object with a force of 12000 N at a direction of [N 15° E]. Monster truck Beta pulls on the object with a force of 15000 N at a direction of [E 5° S]. At what direction (and with what force) should Monster truck Gamma pull on the object so that the three are in a state of equilibrium?

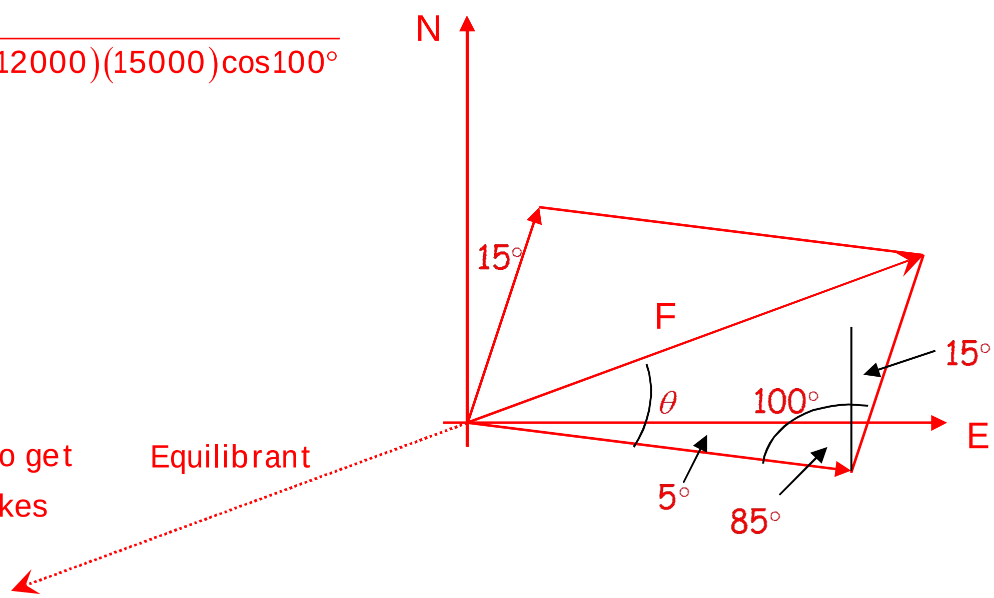
$$|\vec{F}| = \sqrt{12000^2 + 15000^2 - 2(12000)(15000)\cos 100^\circ}$$

$$|\vec{F}| \doteq 20773 \text{ N}$$

$$\frac{\sin\theta}{12000} = \frac{\sin 100^\circ}{20773}$$

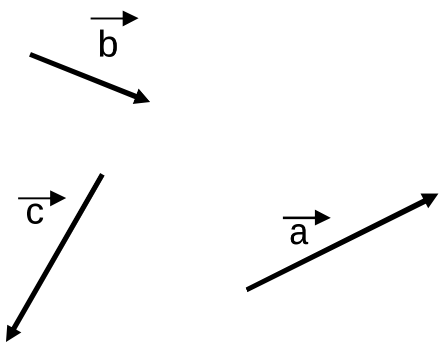
$$\sin\theta \doteq 0.5689$$

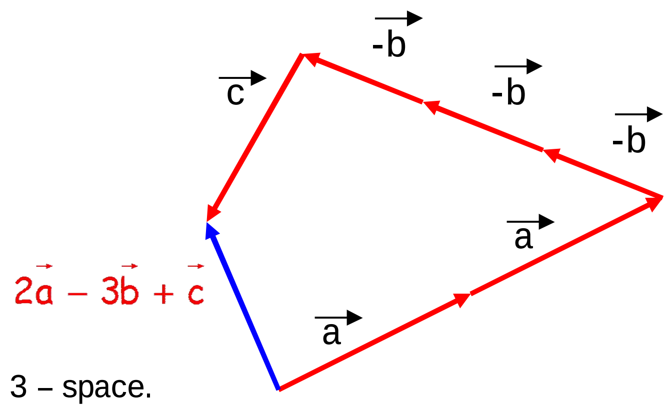
$\theta \doteq 35^\circ$ and $35^\circ - 5^\circ = 30^\circ$ to get the angle the net force makes with the East axis



The Monster truck Gamma should pull with a force of 20773 N at [W 30° S].

3. Use a diagram to draw $2\vec{a} - 3\vec{b} + \vec{c}$ (to scale) given the vectors shown.





4. Prove $\vec{a} \bullet \vec{b} = \vec{b} \bullet \vec{a}$ algebraically in 3 – space.

Let $\vec{a} = [a_1, a_2, a_3]$ and $\vec{b} = [b_1, b_2, b_3]$

$\vec{a} \bullet \vec{b} = [a_1, a_2, a_3] \bullet [b_1, b_2, b_3]$ and $\vec{b} \bullet \vec{a} = [b_1, b_2, b_3] \bullet [a_1, a_2, a_3]$

$= a_1b_1 + a_2b_2 + a_3b_3$ $\qquad$ $= b_1a_1 + b_2a_2 + b_3a_3$

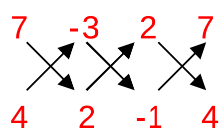
$\qquad = a_1b_1 + a_2b_2 + a_3b_3$

$\therefore\ \vec{a} \bullet \vec{b} = \vec{b} \bullet \vec{a}$ in $\mathbb{R}^3$

5. Find a vector perpendicular to $\vec{b} = [2, 7, -3]$ and $\vec{c} = [-1, 4, 2]$.

$\vec{b} \times \vec{c} = [14 + 12, 3 - 4, 8 + 7]$

$= [26, -1, 15]$



6. Describe the difference between the geometric and the algebraic representation of a vector.

The geometric representation for a vector uses a directed line segment (or ray), while the algebraic representation uses numbers that are the distances horizontally and vertically from the tail to the head (or in the directions of the three coordinate axis in R^3).

7. Find parametric and symmetric equations for the line through the points (7, 0, 5) and (-2, 4, 3).

$\vec{d} = [7 - -2, 0 - 4, 5 - 3] = [9, -4, 2]$ is direction vector for this line.

Parametric Equations:

$x = 7 + 9t$ $\qquad$ $x = -2 + 9t$

$y = -4t$ $\qquad$ or $\qquad$ $y = 4 - 4t$

$z = 5 + 2t$ $\qquad$ $z = 3 + 2t$

Symmetric Equations:

$$\frac{x - 7}{9} = \frac{y}{-4} = \frac{z - 5}{2} \quad \text{or} \quad \frac{x + 2}{9} = \frac{y - 4}{-4} = \frac{z - 3}{2}$$

8. Find vector and Cartesian equations of the plane through the points (11, -2, 3), (8, -4, 1), and (6, 1, 5).

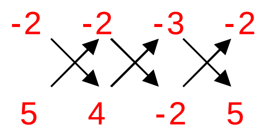
$\overrightarrow{d_1} = [8-11, -4+2, 1-3] = [-3, -2, -2]$

$\overrightarrow{d_2} = [6-8, 1+4, 5-1] = [-2, 5, 4]$

Vector Equation: $[x, y, z] = [11, -2, 3] + t[-3, -2, -2] + s[-2, 5, 4]$

$\overrightarrow{d_1} \times \overrightarrow{d_2} = [-8+10, 4+12, -15-4]$

$= [2, 16, -19]$



Cartesian Equation: $2x + 16y - 19z + D = 0$

$2(11) + 16(-2) - 19(3) + D = 0$

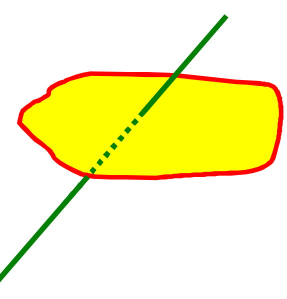
$22 - 32 - 57 + D = 0$

$\therefore D = 67$

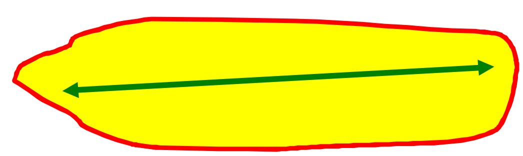
A Cartesian equation is 2x + 16y – 19z + 67 = 0

9. Describe all the ways a line and a plane may exist in 3-Space and the types of intersections they may or may not have.

a) A line may intersect a plane in a unique point.



b) A line may lie on the plane. All points on the line are also on the plane. The line is said to be coincident with the plane.



c) The line may be parallel and distinct from the plane, sharing no common points with the plane.

10. Find the intersections of the following 3 planes. If the solution is infinite, also find one particular solution. Make sure you describe the type of solution. All work **MUST** be shown. You may use a calculator to check your work, but your methodology has to be demonstrated in your solution.

$$x + 3y + 3z - 8 = 0$$
$$x - y + 3z - 4 = 0$$
$$2x + 6y + 6z - 16 = 0$$

$$x + 3y + 3z = 8$$
$$\underline{x - y + 3z = 4}$$
Subtract $4y = 4$
$$\therefore y = 1$$

$2 \times (x + 3y + 3z = 16)$ ---->$2x + 6y + 6z = 16$
$2x + 6y + 6z = 16$ ---------->$\underline{2x + 6y + 6z = 16}$
Subtracting $0 = 0$
This implies an infinite number of solutions.

$x + 3y + 3z - 8 = 0$
Let $z = t$
$x + 3 + 3t = 8$
$\therefore x = 5 - 3t$

These planes intersect in the line $x = 5 - 3t$.
$y = 1$
$z = t$

A particular solution (let t = 1) is $x = 5 - 3(1) = 2$. Therefore (2,1,1) is on all 3 planes.
$y = 1$
$z = 1$

11. What is an average rate of change? Give an example.
An average rate of change is a rate of change over a horizontal distance or period of time that is between two points. It is defined as the amount of vertical change (dependent variable) divided by the horizontal change (independent variable). For example, the average speed of an object between 2 and 5 seconds. Here the period of time is 3 seconds so this would be called an average rate of change.

12. Find the simplified average rate of change of the function $f(x) = -3x^2 + 5x - 4$ between where x = 3 and x = 3 + h.

$$\text{Av Rate} = \frac{f(x+h) - f(x)}{h}$$

$$\text{Av Rate} = \frac{f(3+h) - f(3)}{h}$$

$$\text{Av Rate} = \frac{\left[-(3+h)^2 + 5(3+h) - 4\right] - \left[-(3)^2 + 5(3) - 4\right]}{h}$$

$$\text{Av Rate} = \frac{\left[-9 - 6h - h^2 + 15 + 5h - 4\right] - \left[-9 + 15 - 4\right]}{h}$$

$$\text{Av Rate} = \frac{-h^2 - h + 2 - 2}{h}$$

$$\text{Av Rate} = \frac{-h^2 - h}{h}$$

$$\therefore \text{Av Rate} = -h - 1$$

13. The weasel population (in hundreds) in an area is modeled for the next 8 years by the function $P(t) = -2t^2 + 16t + 5$.

a) What is the instantaneous rate of change of the population at 2 years?

b) What is happening to the population at 2 years? What do you think might be causing this change in the population?

a) $$P'(t) = \lim_{h \to 0} \frac{P(t+h) - P(t)}{h}$$

$$P'(t) = \lim_{h \to 0} \frac{-2(t+h)^2 + 16(t+h) + 5 - \left(-2t^2 + 16t + 5\right)}{h}$$

$$P'(t) = \lim_{h \to 0} \frac{-2t^2 - 4th - 2h^2 + 16t + 16h + 5 + 2t^2 - 16t - 5}{h}$$

$$P'(t) = \lim_{h \to 0} \frac{-4th - 2h^2 + 16h}{h}$$

$$P'(t) = \lim_{h \to 0} -4t - 2h + 16$$

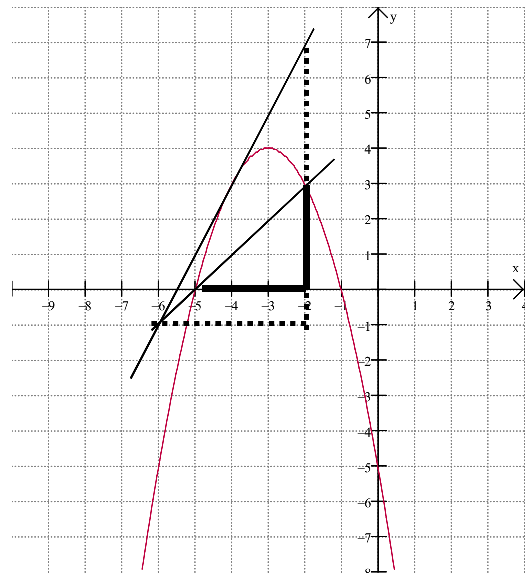
$$P'(t) = -4t + 16$$

$$P'(2) = -4(2) + 16$$

$$P'(2) = 8 \text{ hundred weasels/year}$$

b) At 2 years the population is increasing by 800 weasels per year. This might be caused by the local predator population has declined as part of a normal cycle.

14. Find the following rates of change using a graphical method (using the graph at the right).
   a) The average rate of change from x = -5 to x = -2.
   b) The instantaneous rate of change at x = -4.



a) $Rate_{x=-5 \text{ to } x=-2} = \frac{3}{3} = 1$

b) $Rate_{x=-4} = \frac{8}{4} = 2$

15. Using the first principles definition of the derivative find
   a) $f'(x)$ if $f(x) = 2x^2 + 7x - 3$
   b) $h'(x)$ if $h(x) = 9^x$
   c) $g'(-1)$ if $g(x) = -\frac{5}{2x+3}$

a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h \to 0} \frac{2(x+h)^2 + 7(x+h) - 3 - [2x^2 + 7x - 3]}{h}$

$f'(x) = \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 + 7x + 7h - 3 - 2x^2 - 7x + 3}{h}$

$f'(x) = \lim_{h \to 0} \frac{4xh + 2h^2 + 7h}{h}$

$f'(x) = \lim_{h \to 0} (4x + 2h + 7)$

$\therefore f'(x) = 4x + 7$

b) $h'(x) = \lim_{s \to 0} \frac{h(x+s) - h(x)}{s}$

$h'(x) = \lim_{s \to 0} \frac{9^{x+s} - 9^x}{s}$

$h'(x) = 9^x \lim_{s \to 0} \frac{9^s - 1}{s}$

$\therefore h'(x) = 9^x \ln(9)$

c) $g'(x)=\lim_{h\to 0}\dfrac{g(x+h)-g(x)}{h}$

$$g'(x)=\lim_{h\to 0}\frac{-\dfrac{5}{2(x+h)+3}-\left(-\dfrac{5}{2x+3}\right)}{h}$$

$$g'(x)=\lim_{h\to 0}\frac{-\dfrac{5}{2x+2h+3}+\dfrac{5}{2x+3}}{h}$$

$$g'(x)=\lim_{h\to 0}\frac{\dfrac{-5(2x+3)}{(2x+2h+3)(2x+3)}+\dfrac{5(2x+2h+3)}{(2x+3)(2x+2h+3)}}{h}$$

$$g'(x)=\lim_{h\to 0}\frac{\dfrac{-10x-15+10x+10h+15}{(2x+2h+3)(2x+3)}}{h}$$

$$g'(x)=\lim_{h\to 0}\frac{\dfrac{10h}{(2x+2h+3)(2x+3)}}{h}$$

$$g'(x)=\lim_{h\to 0}\frac{10h}{(2x+2h+3)(2x+3)}\times\frac{1}{h}$$

$$g'(x)=\lim_{h\to 0}\frac{10}{(2x+2h+3)(2x+3)}$$

$$\therefore g'(x)=\frac{10}{(2x+3)^2}$$

16. A certain bacteria culture starts with a population of 120. The population grew to 5000 in 1 day.

a) Find a formula for the number of bacteria after t hours.

b) How many bacteria were there after 14 hours?

a) $P(t)=120e^{kt}$

$5000=120e^{k24}$

$\dfrac{125}{3}=e^{24k}$

$24k=\ln\left(\dfrac{125}{3}\right)$

$k=\dfrac{\ln\left(\dfrac{125}{3}\right)}{24}\doteq 0.1554$

The formula is $P(t)=120e^{0.1554t}$

b) $P(t)=120e^{0.1554t}$

$P(14)=120e^{0.1554(14)}$

$P(14)=1056.896$

After 14 hours there were about 1057 bacteria.

17. A sample of radioactive Plutonium started with a mass of 16.2 mg. This isotope's half-life is 2.7 hours.
    a) Find a formula for the amount of Plutonium remaining after t hours.
    b) How long (from the original sample) will it take for the sample size to be down to only 1 mg?

Solution 1:

a) $N(t) = 16.2e^{kt}$

$8.1 = 16.2e^{k(2.7)}$

$\frac{8.1}{16.2} = \frac{16.2e^{2.7k}}{16.2}$

$0.5 = e^{2.7k}$

$2.7k = \ln 0.5$

$k = \frac{\ln 0.5}{2.7} \doteq -0.2567$

Formula: $N(t) = 16.2e^{-0.2567t}$

b) $N(t) = 16.2e^{-0.2567t}$

$1 = 16.2e^{-0.2567t}$

$\frac{1}{16.2} = \frac{16.2e^{-0.2567t}}{16.2}$

$\frac{1}{16.2} = e^{-0.2567t}$

$-0.2567t = \ln\left(\frac{1}{16.2}\right)$

$t = -\frac{\ln\left(\frac{1}{16.2}\right)}{0.2567}$

$\therefore t \doteq 10.85$ hours

Solution 2:

a) $N(t) = 16.2\left(2^{-\frac{t}{2.7}}\right)$

b) $N(t) = 16.2\left(2^{-\frac{t}{2.7}}\right)$

$1 = 16.2\left(2^{-\frac{t}{2.7}}\right)$

$\frac{1}{16.2} = \frac{16.2}{16.2}\left(2^{-\frac{t}{2.7}}\right)$

$\frac{1}{16.2} = 2^{-\frac{t}{2.7}}$

$-\frac{t}{2.7} = \log_2\left(\frac{1}{16.2}\right)$

$t = -2.7\log_2\left(\frac{1}{16.2}\right)$

$t = 10.85$ hours

18. Find the first derivative for each of the following functions

a) $f(x) = 4x^7 - \sqrt{2}x^3 - 8x + 6$

$f'(x) = 28x^6 - 3\sqrt{2}x^2 - 8$

b) $f(x) = (2x^3 - 5x)^6$

$f'(x) = 6(2x^3 - 5x)^5(6x^2 - 5)$

$f'(x) = 6(6x^2 - 5)(2x^3 - 5x)^5$

c) $y = \dfrac{28x^3 + 14x^2 - 21x}{7x}$

$y = 4x^2 + 2x - 3$

$\dfrac{dy}{dx} = 8x + 2$

d) $f(x) = 7x^5\sqrt{3x^4 - 5x^2}$

$f(x) = 7x^5(3x^4 - 5x^2)^{\frac{1}{2}}$

$f'(x) = 35x^4(3x^4 - 5x^2)^{\frac{1}{2}} + 7x^5\frac{1}{2}(3x^4 - 5x^2)^{-\frac{1}{2}}(12x^3 - 10x)$

$f'(x) = 35x^4\sqrt{3x^4 - 5x^2} + \dfrac{42x^8 - 35x^6}{\sqrt{3x^4 - 5x^2}}$

$f'(x) = 35x^4\sqrt{3x^4 - 5x^2}\,\dfrac{\sqrt{3x^4 - 5x^2}}{\sqrt{3x^4 - 5x^2}} + \dfrac{42x^8 - 35x^6}{\sqrt{3x^4 - 5x^2}}$

$f'(x) = \dfrac{35x^4(3x^4 - 5x^2)}{\sqrt{3x^4 - 5x^2}} + \dfrac{42x^8 - 35x^6}{\sqrt{3x^4 - 5x^2}}$

$f'(x) = \dfrac{105x^8 - 175x^6}{\sqrt{3x^4 - 5x^2}} + \dfrac{42x^8 - 35x^6}{\sqrt{3x^4 - 5x^2}}$

$\therefore f'(x) = \dfrac{147x^8 - 210x^6}{\sqrt{3x^4 - 5x^2}}$

e) $f(x) = \dfrac{5}{(5x^2 + 8)^3}$

$f(x) = 5(5x^2 + 8)^{-3}$

$f'(x) = -15(5x^2 + 8)^{-4}(10x)$

$\therefore f'(x) = -\dfrac{150x}{(5x^2 + 8)^4}$

f) $f\left(x\right)=\sin\left(x^2+1\right)$

$f'\left(x\right)=\cos\left(x^2+1\right)\times 2x$

$f'\left(x\right)=2x\cos\left(x^2+1\right)$

g) $y=4e^{x^3}$

$y'=4e^{x^3}\left(3x^2\right)$

$\therefore y'=12x^2e^{x^3}$

h) $y=\left(e^{2x}\right)\sin\left(4x-5\right)$

$\frac{dy}{dx}=\left(2e^{2x}\right)\sin\left(4x-5\right)+e^{2x}4\cos\left(4x-5\right)$

$\therefore \frac{dy}{dx}=\left(2e^{2x}\right)\sin\left(4x-5\right)+4e^{2x}\cos\left(4x-5\right)$

i) $f\left(x\right)=6^{4x}$

$f'\left(x\right)=6^{4x}\times\ln\left(6\right)\times 4$

$\therefore f'\left(x\right)=4\ln\left(6\right)6^{4x}$

j) $b\left(x\right)=6\cos 4x^2$

$b'\left(x\right)=6\left(-\sin 4x^2\right)\times 8x$

$\therefore b'\left(x\right)=-48x\sin 4x^2$

19. A transportation company sells 2000 bus tickets per day when the price is $10 per ticket. They believe that for every $1 increase in ticket price they will sell 100 tickets less. Use the calculus to determine what price the company should charge to maximize their revenue.

Let x represent the number of $1 increases in ticket price and R(x) be the revenue as a function of the number of increases.

$R\left(x\right)=\left(10+x\right)\left(2000-100x\right)$

$R\left(x\right)=20000-1000x+2000x-100x^2$

$R\left(x\right)=20000+1000x-100x^2$

$R'\left(x\right)=1000-200x$

$1000-200x=0$ $\left[\text{The max occurs where } R'\left(x\right)=0\right]$

$200x=1000$

$\therefore x=5$

If x = 5, then the price is 10 + 5 = $15.00 per ticket. This will yield the highest possible revenue.

20. Find the equation of the tangent line to the function $y = 4x^3 - 5x^2 + 8x$ where $x = 1$.

$y' = 12x^2 - 10x + 8$

$y'_{x=1} = 12(1)^2 - 10(1) + 8$

$y'_{x=1} = 10$

$y = mx + b$

$y = 10x + b$ [need to find y]

$y = 4(1)^3 - 5(1)^2 + 8(1)$

$y = 7$

$7 = 10(1) + b$

$\therefore b = -3$

The equation of the tangent is

$y = 10x - 3$

21. Sketch the graph of the following functions using local maximum/minimums, intercepts, asymptotes, points of inflection and concavity.

a) $f(x) = x^4 - 8x^3 + 18x^2$

Intercepts: $x^4 - 8x^3 + 18x^2 = 0$

$x^2(x^2 - 8x + 18) = 0$

$x^2 = 0$ or $x^2 - 8x + 18 = 0$

$\therefore x = 0$ $\quad x^2 - 8x + 18 = 0$ has no solution

$f'(x) = 4x^3 - 24x^2 + 36x$

$4x^3 - 24x^2 + 36x = 0$

$4x(x^2 - 6x + 9) = 0$

$4x = 0 \quad x^2 - 6x + 9 = 0$

$x = 0 \quad (x-3)^2 = 0$

$x = 3$

$f'(x) = 4x^3 - 24x^2 + 36x$

$f''(x) = 12x^2 - 48x + 36$

$12x^2 - 48x + 36 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$\therefore x = 3, 1$ [possible POI]

$f(1) = (1)^4 - 8(1)^3 + 18(1)^2$

$f(1) = 11$

$f(3) = (3)^4 - 8(3)^3 + 18(3)^2$

$f(3) = 81 - 216 + 162$

$f(3) = 27$

$f''(0) = 12(0)^2 - 48(0) + 36 = 36$

$\therefore x = 0$ is a local min (concave up)

$f''(2) = 12(2)^2 - 48(2) + 36 = -12$

concave down

$f''(3) = 12(3)^2 - 48(3) + 36 = 0$

$f''(4) = 12(4)^2 - 48(4) + 36 = 36$

$x = 3$ is a POI since $f''(x)$ changes sign

Summary:

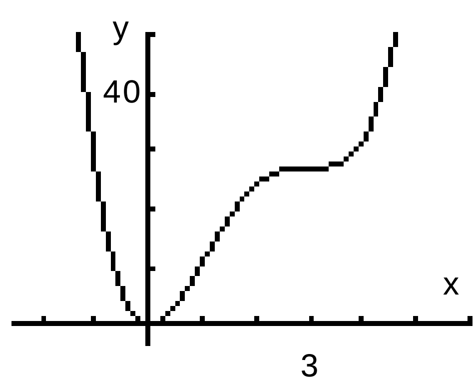
Local min at $(0, 0)$

X-intercept at $(0, 0)$

POI at $(3, 27)$ & $(1, 11)$

Concave Down: $1 < x < 3$

Concave Up: $x < 1$ & $x > 3$



b) $f(x)=\dfrac{10}{(x+2)^2}$

Summary:
Local min/max: none
y-intercept at $(0,2.5)$
POI: none
Concave Up: $x<-2$ & $x>-2$

y-intercept:

$f(0)=\dfrac{10}{(0+2)^2}$

$f(0)=\dfrac{10}{4}$

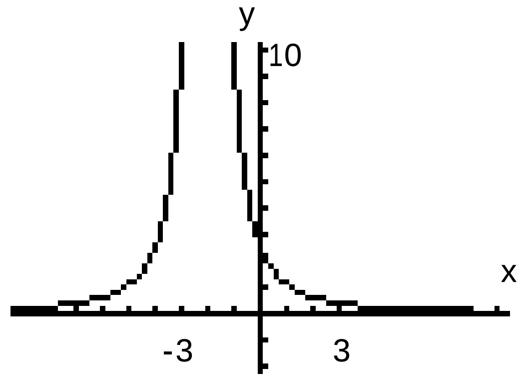
$f(0)=2.5$

No x-intercept since

$\dfrac{10}{(x+2)^2}\neq 0$

Vertical Asymptote at $x=-2$

Since $f(x)=\dfrac{10}{(x+2)^2}>0$, the graph goes up on both sides of the asymptote.



$f(x)=10(x+2)^{-2}$

$f'(x)=-20(x+2)^{-3}$

$f'(x)=-\dfrac{20}{(x+2)^3}$

Since $-\dfrac{20}{(x+2)^3}\neq 0$

No local max/min.

$f'(x)=-20(x+2)^{-3}$

$f''(x)=60(x+2)^{-4}$

$f''(x)=\dfrac{60}{(x+2)^4}$

Since $\dfrac{60}{(x+2)^4}>0$, the graph is entirely concave up.