

MCV4U Practice Exam

Exam Instructions:

1. Answer all questions in the space provided (*none is provided on this practice exam, but there will be room on the "real exam"*).
2. If you need additional paper or graph paper ask for it, otherwise all answers should be written on this exam paper.
3. Only write answers on the front of any page and in dark pencil, black or blue pen.
4. You may use any handheld calculator (including the TI 83 Plus) for this entire exam.
5. The student may use on this exam a cheat sheet that is no larger than an 8.5 in by 5.5 in piece of paper, written on one side only.
6. 10 additional communication marks are overall throughout the exam.

MCV4U eLearning Final Exam Outline

The unit breakdown is as follows:

Unit	Marks
Unit 1 Vectors	15
Unit 2 Equations of Lines and Planes	15
Unit 3 Rates of Change	18
Unit 4 Derivatives of Sinusoidal and Exponential Functions	13
Unit 5 Derivatives	17
Unit 6 Curve Sketching	12
Overall Communication	10
Total Marks	100

There are 20 multiple choice questions in part 1 of the exam covering almost 2 and a half pages (letter size), and then part B has 9 and a half pages of 16 questions with room left to do all work.

Part I: Place your answers for Part I in the space provided.

1. Which of the following is a vector quantity? (____)
A. time B. force C. mass D. temperature E. none of these
2. Using the **Triangle Law of Addition** to add two vectors, the vectors must be drawn (____)
A. tail to head B. head to head C. tail to tail
D. head to tail E. backwards

3. Given the points B(-3, 1, -2) and C(3, -5, 10) the vector \overline{BC} in component form is (____)
 A. [6, -6, 12] B. [-6, 6, -12] C. [-5, -4, 8] D. [1, -1, 2] E. [-1, 1, -2]
4. Given $|\vec{a}| = 5$, $|\vec{b}| = 6$ and the angle between them is 60° , then $\vec{a} \bullet \vec{b} =$ (____)
 A. 15 B. 30 C. 51.96 D. 5.5 E. none of these
5. If $\vec{d} \times \vec{h} = [2, -6, 7]$, then $\vec{h} \times \vec{d} =$ (____)
 A. [2, -6, 7] B. [7, -6, 2] C. [-2, 6, -7] D. [-6, 7, 2] E. none of these
6. Which of the following are parametric equations of a line through (3,-2) and (9,-4)? (____)
 A. $x = 3 + 9t$ B. $x = 3 - 9t$ C. $x = -3 + 6t$ D. $x = 9 + 3t$ E. none of these
 $y = -2 - 4t$ $y = -2 + 4t$ $y = +2 - 2t$ $y = -4 - t$
7. Which of the following points is on the plane $3x + y - 4z - 5 = 0$? (____)
 A. (3, 1 -4) B. (2, -1, 0) C. (1, -3, -1) D. (-3, 7, 2) E. none of these
8. Which of the following yields a vector perpendicular to both original vectors? (____)
 A. dot product B. scalar product C. cross product E. none of these
9. The derivative of the function $y = f(x)$ where $x = 3$ is (____)
 A. $\lim_{\Delta x \rightarrow 0} \frac{f(x+a) - f(x)}{\Delta x}$ B. $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
 C. $\lim_{h \rightarrow 0} \frac{f(x+3) - f(x)}{h}$ D. $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$ E. none of these
10. The Power Differentiation Rule is (____)
 A. $\frac{d}{dx}(x^n) = nx^{n-1}$ B. $\frac{d}{dx}(k) = 0, k \in R$
 C. $\frac{d}{dx}(u^n) = nu^{n-1} \frac{du}{dx}$, if u is a function of x
 D. $\frac{d}{dx}(u^n) = nu^{n-1}$ E. none of these

11. If $f(x) = r(x)h(x)$ then $f'(x) =$ (____)
- A. $r(x)h'(x) - h(x)r'(x)$ B. $r'(x)h'(x)$
 C. $h(x)r'(x) - r(x)h'(x)$ D. $r'(x)h(x) + h'(x)r(x)$ E. none of these

12. If $y = f(g(x))$, then $\frac{dy}{dx} =$ (____)
- A. $f(g'(x))$ B. $f(g'(x))g(x)$ C. $f'(g(x))$
 D. $f'(g'(x))$ E. none of these

13. The graph of the function $y = f(x)$ is always concave up where (____)
- A. $f'(x) = 0$ B. $f''(x) = 0$ C. $f''(x) < 0$
 D. $f''(x) > 0$ E. $f'(x) \leq 0$

14. $\frac{d}{dx}[2e^{3x}] =$ (____)
- A. $2e^{3x}$ B. e^{3x} C. $2e^x$ D. $6e^{3x}$ E. none of these

15. If $y = \sin 5x$, then $y' =$ (____)
- A. $-\cos 5x$ B. $5 \cos x$ C. $-5 \cos 5x$ D. $5 \cos 5x$ E. none of these

16. What kind of line shows an instantaneous rate of change the best? (____)
- A. a secant line B. a tangent line

17. $\frac{d}{dx}[5^{4x^2}] =$ (____)
- A. $8x5^{4x^2}$ B. $8x(\ln 5)5^{4x^2}$ C. $4x^25^{4x^2-1}$
 D. $8x5^{4x^2-1}$ E. $(\ln 5)5^{8x}$

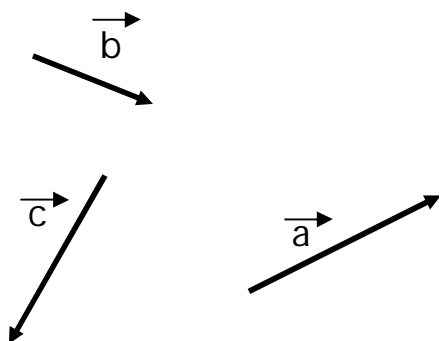
18. An interval upon which a function has a negative derivative it is said to be _____. (____)
- A. concave up B. increasing C. concave down
 D. decreasing E. none of these

19. An interval upon which a function has a positive second derivative the function is said to be _____ (____)
- A. concave up B. increasing C. concave down
 D. decreasing E. none of these

20. Which of the following is **not** a proper symbol for the derivative? (____)
- A. y' B. $\frac{dy}{dx}$ C. $\frac{d}{dx}$ D. D_x E. $h'(x)$

Part II: Place your answers for Part II in the space provided on this exam paper. If you run out of room for any question, additional work can be shown on foolscap or lined paper.

- Given the vectors $\vec{p} = [3, -1, 4]$ and $\vec{u} = [-2, 5, 1]$. Find the angle between the vectors \vec{p} and \vec{u} .
- Monster truck Alpha pulls on an object with a force of 12000 N at a direction of [N 15° E]. Monster truck Beta pulls on the object with a force of 15000 N at a direction of [E 5° S]. At what direction (and with what force) should Monster truck Gamma pull on the object so that the three are in a state of equilibrium?
- Use a diagram to draw $2\vec{a} - 3\vec{b} + \vec{c}$ (to scale) given the vectors shown.



- Prove $\vec{a} \bullet \vec{b} = \vec{b} \bullet \vec{a}$ algebraically in 3 - space.
- Find a vector perpendicular to $\vec{b} = [2, 7, -3]$ and $\vec{c} = [-1, 4, 2]$.
- Describe the difference between the geometric and the algebraic representation of a vector.

7. Find parametric and symmetric equations for the line through the points $(7, 0, 5)$ and $(-2, 4, 3)$.
8. Find vector and Cartesian equations of the plane through the points $(11, -2, 3)$, $(8, -4, 1)$, and $(6, 1, 5)$.
9. Describe all the ways a line and a plane may exist in 3-Space and the types of intersections they may or may not have.
10. Find the intersections of the following 3 planes. If the solution is infinite, also find one particular solution. Make sure you describe the type of solution. All work **MUST** be shown. You may use a calculator to check your work, but your methodology has to be demonstrated in your solution.

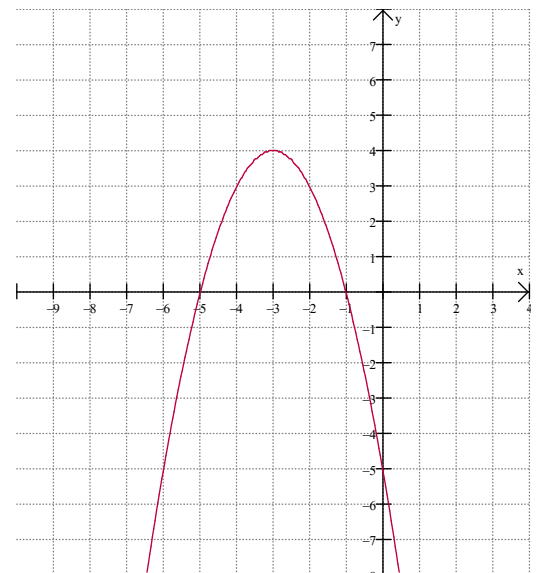
$$x + 3y + 3z - 8 = 0$$

$$x - y + 3z - 4 = 0$$

$$2x + 6y + 6z - 16 = 0$$

11. What is an average rate of change? Give an example.
12. Find the simplified average rate of change of the function $f(x) = -3x^2 + 5x - 4$ between where $x = 3$ and $x = 3 + h$.
13. The weasel population (in hundreds) in an area is modeled for the next 8 years by the function $P(t) = -2t^2 + 16t + 5$.
 - a) What is the instantaneous rate of change of the population at 2 years?
 - b) What is happening to the population at 2 years? What do you think might be causing this change in the population?

14. Find the following rates of change using a graphical method (using the graph at the right).
 - a) The average rate of change from $x = -5$ to $x = -2$.
 - b) The instantaneous rate of change at $x = -4$.



15. Using the first principles definition of the derivative find

a) $f'(x)$ if $f(x) = 2x^2 + 7x - 3$

b) $h'(x)$ if $h(x) = 9^x$

c) $g'(-1)$ if $g(x) = -\frac{5}{2x+3}$

16. A certain bacteria culture starts with a population of 120. The population grew to 5000 in 1 day.

a) Find a formula for the number of bacteria after t hours.

b) How many bacteria were there after 14 hours?

17. A sample of radioactive Plutonium started with a mass of 16.2 mg. This isotope's half-life is 2.7 hours.

a) Find a formula for the amount of Plutonium remaining after t hours.

b) How long (from the original sample) will it take for the sample size to be down to only 1 mg?

18. Find the first derivative for each of the following functions

a) $f(x) = 4x^7 - \sqrt{2}x^3 - 8x + 6$

b) $f(x) = (2x^3 - 5x)^6$

c) $y = \frac{28x^3 + 14x^2 - 21x}{7x}$

d) $f(x) = 7x^5 \sqrt{3x^4 - 5x^2}$

e) $f(x) = \frac{5}{(5x^2 + 8)^3}$

f) $f(x) = \sin(x^2 + 1)$

g) $y = 4e^{x^3}$

h) $y = (e^{2x})\sin(4x - 5)$

i) $f(x) = 6^{4x}$

j) $b(x) = 6\cos 4x^2$

19. A transportation company sells 2000 bus tickets per day when the price is \$10 per ticket. They believe that for every \$1 increase in ticket price they will sell 100 tickets less. Use the calculus to determine what price the company should charge to maximize their revenue.

20. Find the equation of the tangent line to the function $y = 4x^3 - 5x^2 + 8x$ where $x = 1$.

21. Sketch the graph of the following functions using local maximum/minimums, intercepts, asymptotes, points of inflection and concavity.

a) $f(x) = x^4 - 8x^3 + 18x^2$

b) $f(x) = \frac{10}{(x+2)^2}$