

2020 MCF3M FINAL EXAM REVIEW

Quadratic Equations

1. Solve each of the following quadratic equations using the best method: (Factoring, completing the square, quadratic formula)

a) $x^2 + 2x - 15 = 0$

b) $2x^2 - x = 15$

c) $4x^2 - 16x = 0$

d) $10x^2 - 2x = 5 - 25x$

e) $\frac{3}{2}x^2 = 12x - 6$

f) $6x^2 - 5x = 1$

g) $3x^2 + x = 5$

h) $5x^2 + 20 = 0$

2. Determine the optimal value (maximum or minimum) in each case and at what value of x it occurs:

a) $f(x) = x^2 - 16x + 24$

b) $f(x) = -4x^2 + 24x - 17$

c) $f(x) = \frac{1}{2}x^2 - 11x + 5$

3. The height of the ball, h, in metres, can be modeled by the function $h = -4t^2 + 8t + 1$, where t is time in seconds.

a) What is the height of the ball after 1 second?

b) What is the maximum height of the basketball?

c) At what time does the basketball reach its maximum height?

4. The height of a firework, h, in metres, is given by $h = -5t^2 + 30t + 3$, where t is the time in seconds, after the firework has been fired.

a) At what height was the firework fired?

b) For how many seconds was the rocket in the air?

c) Determine the maximum height of the rocket.

5. A rectangular lot is 15 m longer than it is wide. If the area is 1000 m^2 , determine the length.

6. Determine the equations of the following parabolas for the given information.

a) vertex (0,0) passing through the point (-2,1)

b) vertex $(3, -\frac{1}{2})$ and y-intercept 6

c) axis of symmetry $x = 2$, with a minimum value of 3, with the same shape as $y = x^2$

1a) $x = -5$ or $x = 3$ b) $x = -5/2$ or $x = 3$ c) $x = 0$ or $x = 4$ d) $x = 1/5$ or $x = -5/2$ e) $x = 4 \pm 2\sqrt{3}$ f)

$x = 1$ or $x = -1/6$ g) $x = (-1 \pm \sqrt{61})/6$ h) not possible 2a)

Functions and Transformations

1. For each of the following:

i) State if each represents a function, and explain your reasoning in each case.

a) $f(x) = x - 2$

b) $f(x) = x^2 - 4x + 2$

2. If $f(x) = x^2 - 4x - 2$ and $g(x) = 2x - 1$, determine:

a) $f(-2)$

b) $g(f(x))$

c) $f(g(x))$

d) x when $g(x) = -1$

3. Describe the transformations on $f(x) = x^2$ that give the following functions:

a) $f(x) = -(x + 2)^2 - 1$

b) $f(x) = \frac{1}{3}(x - 1)^2 + 3$

4. Write the transformed equation if $f(x) = x^2$ undergoes the following transformation: a vertical stretch of factor 2, shifted up 3 units, reflected in the x-axis and shifted to the right 5 units.

5. Graph each of the following parabolas, and state the domain and range for each:

a) $f(x) = 2x^2 + 1$

b) $f(x) = -\frac{1}{2}(x - 3)^2$

c) $f(x) = -3(x + 2)^2 + 1$

Exponent Laws

1. Evaluate without a calculator. Show all steps. (No decimals)

a) $(-3)^{-2}$ b) $\frac{4^0 + 2^0}{3^{-1}}$ c) $8^{\frac{2}{3}}$ d) $-9^{\frac{3}{2}}$ e) $\left(\frac{-8}{125}\right)^{\frac{2}{3}}$

f) $\frac{3^2 - 2^3}{2^{-2} - 3^{-1}}$ g) $\left(\sqrt[3]{27} + 64^{\frac{1}{3}}\right)^2$ h) $\frac{9^0}{3^{-2} - 2^{-3}}$

2. Given $\left(\frac{3a^2b^3}{5ab^2}\right)^2$

a) Simplify the expression

b) Evaluate the simplified expression if $a = -2$ and $b = 1$.

3. Simplify:

a) $(-3x^2y^3z)^2(2x^3yz^2)^3$ b) $\left(\frac{12x^{10}y^9}{-4x^9y^{-1}}\right)\left(\frac{22x^{19}y}{11x^{17}y^{-3}}\right)$ c) $(81x^6y^8)^{\frac{1}{2}}(8x^3y^6)^{\frac{1}{3}}$

4. Solve the following exponential equations.

a) $2^x = 32$

b) $5 \times 25^x = 1$

c) $3^{2n-1} \times 27^{n-2} = 81^{3n}$

d) $7^{2x+3} = \frac{1}{49}$

Expanding and Factoring Polynomials

1. Simplify.

a) $(3x - 2)^2$

b) $(2x + 1)(2x - 1)$

c) $-(x - 2)^2 + (x + 1)(3x + 2)$

d) $(2x + 5)^2 - (3x - 2)^2$

e) $(7x^2 - 2x + 1) - (5x^2 + 3x - 2)$

f) $3m(2 + 2m(2 - m)) + m(2(2m - 1) - 2)$

g) $(2y + 1)(5y^2 + 3y - 2)$

h) $(5b^2 - 3b + 2)^2$

2. Factor the following:

a) $48a^8b^7c^{10} - 24a^6b^{10}c^7$

b) $x^2 - x - 30$

c) $m^2 - 11m + 30$

d) $x^2 - 7x - 30$

e) $x^2 - 20x + 100$

f) $x^2 - x - 90$

g) $6x^2 + 5x - 6$

h) $9m^2 - 1$

i) $6x^2 + 17xy + 12y^2$

j) $6x^3 + 26x^2 - 20x$

k) $128x^2y - 50y^3$

l) $\frac{36x^2}{49} - \frac{121y^2}{169}$

m) $49 - 42x + 9x^2$

n) $625x^8y^4 - 16z^8$

o) $1 - 6x + 9x^2 - 4y^2$

Trigonometry

1. Solve each of the following triangles. Round your answer to 1 decimal place.
 - a) $\triangle ABC$, $\angle B = 90^\circ$, $\angle C = 15^\circ$, $a = 10\text{ cm}$
 - b) $\triangle STU$, $\angle S = 63^\circ$, $\angle T = 90^\circ$, $u = 9\text{ cm}$
 - c) $\triangle PQR$, $\angle Q = 90^\circ$, $\angle P = 31^\circ$, $p = 5\text{ cm}$
 - d) $\triangle CDE$, $c = 30\text{ cm}$, $d = 42\text{ cm}$, $\angle D = 33^\circ$
 - e) $\triangle FGH$, $\angle F = 31.5^\circ$, $\angle G = 73.8^\circ$, $h = 22.3\text{ cm}$

2. A tree casts a shadow 30 m long when the sun's rays are at an angle of 36° to the ground. How tall is the tree?

2. Two office towers are 25 m apart. From the 12th floor (20 metres up) of the shorter tower, the angle of elevation to the top of the other tower is 70° . Find the height of the taller tower.

3. A hockey net is 1.83 m wide. A player shoots from a point where the puck is 12 m from one goal post and 10.8 m from the other. Within what angle must he shoot?

4. From the top of a lighthouse, Patti sees a whale in difficulty. The angle of depression to the whale is 34° . From another observation point 15 metres directly below, Savanga sees the same whale with an angle of depression of 17° .
 - a) Draw and label a diagram that illustrates the situation described above.
 - b) How far is the whale from the foot of the observation tower?

5. Graph the following trigonometric functions where $-360^\circ \leq x \leq 360^\circ$, and state the domain and range of each:
 - a) $f(x) = 2\sin(x - 90^\circ) + 1$
 - b) $f(x) = -3\sin(x + 45^\circ)$

Exponential Functions

1. Graph each of the following using a table of values. State the domain and range for each:

a) $f(x) = 2^x$ b) $f(x) = \left(\frac{1}{3}\right)^x$

2. The population of a strain of bacteria doubles every 40 minutes. If there are initially 500 bacteria:
- Write a formula that represents the number of bacteria after t hours.
 - Determine the amount of bacteria after 2 hours.
 - After how many hours will there be 50 000 bacteria?
3. The population of a city decreases at a rate of 0.5% per year. If the population in 2008 is 550 000, determine the projected population of the city in 2018.
4. A vintage car is purchased for \$35700. Its value appreciates by 7% each year.
- Write a formula that represents the value of the car after n years.
 - Use your formula to determine the value of the car after 6 years.
 - Determine when the value of the car will double.
5. A radioactive isotope has a half life of 12 hours. Suppose you have 200 g of the isotope today. Determine the mass of the isotope after 142 minutes.

Finance

1. Determine:
 - a) the amount \$5000 will grow to if invested at 2.5% per annum, compounded monthly for 4 years.
 - b) the principal that must be invested now at 3.6% per annum, compounded semi-annually to be worth \$4 000 in 6 years.
 - c) The total accumulated amount of \$50 invested every week at 1.7% per annum, compounded weekly for 10 years.
2. In order to have \$10000 for a used car in 4 years, you plan to deposit money into a savings account every month that pays 2.4% per annum, compounded monthly,
 - a) What will your regular deposits be?
 - b) How much interest would you have earned?
3. Emma plans to withdraw \$600 every month from her retirement fund. How much should she invest into an account today that pays 3.5% per annum, compounded monthly so that she withdraw this amount for the next 10 years?
4. How long will it take, to the nearest year, for \$8000 to grow to \$10000 in an account that pays 2.5% per annum, compounded semi-annually?
5. Carla deposits \$1000 every 3 months in an account that pays 1.4% per annum, compounded quarterly. After 4 years she stops depositing and lets her money accumulate interest (at the same interest rate) for another 5 years. How much money does Carla end up with in her account?