Proving some Reduction Formulas

Date

Reduction Formulas for Sine and Cosine

$$\int \sin^n x\,dx = \frac{-1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx \qquad \int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

Proof #1:

Prove: $\int \sin^n x\,dx = \frac{-1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$

Let $u = \sin^{n-1} x$ then $du = (n-1)\sin^{n-2} x \cdot \cos x\,dx$

$dv = \sin x\,dx$ then $v = -\cos x$

$$\int \sin^n x\,dx$$
$$= \int \sin^{n-1} x \cdot \sin x\,dx$$
$$= uv - \int v\,du$$
$$= \sin^{n-1} x(-\cos x) - \int (-\cos x)(n-1)\sin^{n-2} x \cdot \cos x\,dx$$
$$= -\sin^{n-1} x\cos x + \int \cos^2 x(n-1)\sin^{n-2} x\,dx$$
$$= -\sin^{n-1} x\cos x + (n-1)\int (1-\sin^2 x)\sin^{n-2} x\,dx$$
$$= -\sin^{n-1} x\cos x + (n-1)\int (\sin^{n-2} x - \sin^n x)dx$$
$$= -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\,dx - (n-1)\int \sin^n x\,dx$$

Bring the last term to left side

$$\int \sin^n x\,dx + (n-1)\int \sin^n x\,dx = -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\,dx$$
$$n\int \sin^n x\,dx = -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\,dx$$
$$\int \sin^n x\,dx = \frac{-1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

Proof #2:

Prove: $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$

Let $u = \cos^{n-1} x$ then $du = (n-1)\cos^{n-2} x \cdot -\sin x\,dx$

$dv = \cos x\,dx$ then $v = \sin x$

$$\int \cos^n x\,dx$$
$$= \int \cos^{n-1} x \cdot \cos x\,dx$$
$$= uv - \int v\,du$$
$$= \cos^{n-1} x(\sin x) - \int (\sin x)(n-1)\cos^{n-2} x \cdot (-\sin x)dx$$
$$= \cos^{n-1} x\sin x + \int \sin^2 x(n-1)\cos^{n-2} x\,dx$$
$$= \cos^{n-1} x\sin x + (n-1)\int (1-\cos^2 x)\cos^{n-2} x\,dx$$
$$= \cos^{n-1} x\sin x + (n-1)\int (\cos^{n-2} x - \cos^n x)dx$$
$$= \cos^{n-1} x\sin x + (n-1)\int \cos^{n-2} x\,dx - (n-1)\int \cos^n x\,dx$$

Bring the last term to left side

$$\int \cos^n x\,dx + (n-1)\int \cos^n x\,dx = \cos^{n-1} x\sin x + (n-1)\int \cos^{n-2} x\,dx$$
$$n\int \cos^n x\,dx = \cos^{n-1} x\sin x + (n-1)\int \cos^{n-2} x\,dx$$
$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

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Reduction Formulas for $Sin^m x\ Cos^n x\ \ (m < n)$

$$\int \sin^m x \cos^n x dx = -\frac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2} x \cos^n x dx \quad (m < n)$$

Proof #3:

Prove: $\int \sin^m x \cos^n x dx = -\frac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2} x \cos^n x dx \quad (m < n)$

$\int \sin^m x \cos^n x dx$

$= \int \sin^{m-1} x \cos^n x \sin x dx$

$= uv - \int v du$

Let $u = \sin^{m-1} x \cos^n x$

then $du = \left[(m-1)\sin^{m-2} x \cdot \cos x\right] \cdot \cos^n x + \sin^{m-1} x \cdot \left[n \cos^{n-1} x \cdot (-\sin x)\right] dx$

$= \left[(m-1)\sin^{m-2} x \cdot \cos^{n+1} x - n \sin^m x \cdot \cos^{n-1} x\right] dx$

$dv = \sin x dx$ then $v = -\cos x$

$$= \left(\sin^{m-1} x \cos^n x\right) \cdot (-\cos x) - \int (-\cos x)\left[(m-1)\sin^{m-2} x \cdot \cos^{n+1} x - n \sin^m x \cdot \cos^{n-1} x\right] dx$$

$$= \left(-\sin^{m-1} x \cos^{n+1} x\right) + \int (\cos x)\left[(m-1)\sin^{m-2} x \cdot \cos^{n+1} x - n \sin^m x \cdot \cos^{n-1} x\right] dx$$

$$= \left(-\sin^{m-1} x \cos^{n+1} x\right) + \int \left[(m-1)\sin^{m-2} x \cdot \cos^{n+2} x - n \sin^m x \cdot \cos^n x\right] dx$$

$$= \left(-\sin^{m-1} x \cos^{n+1} x\right) + (m-1)\int \left[\sin^{m-2} x \cdot \cos^{n+2} x\right] dx - n\int \sin^m x \cos^n x dx$$

$$= \left(-\sin^{m-1} x \cos^{n+1} x\right) + (m-1)\int \left[\sin^{m-2} x \cdot \cos^n x \cdot \cos^2 x\right] dx - n\int \sin^m x \cos^n x dx$$

$$= \left(-\sin^{m-1} x \cos^{n+1} x\right) + (m-1)\int \left[\sin^{m-2} x \cdot \cos^n x \cdot \left(1 - \sin^2 x\right)\right] dx - n\int \sin^m x \cos^n x dx$$

$$= \left(-\sin^{m-1} x \cos^{n+1} x\right) + (m-1)\int \left[\sin^{m-2} x \cdot \cos^n x - \sin^m x \cdot \cos^n x\right] dx - n\int \sin^m x \cos^n x dx$$

$$= \left(-\sin^{m-1} x \cos^{n+1} x\right) + (m-1)\int \left[\sin^{m-2} x \cdot \cos^n x\right] dx - (m-1)\int \left[\sin^m x \cdot \cos^n x\right] dx - n\int \sin^m x \cos^n x dx$$

Bring the last two terms to left side

$$\int \sin^m x \cos^n x dx + (m-1)\int \left[\sin^m x \cdot \cos^n x\right] dx + n\int \sin^m x \cos^n x dx = \left(-\sin^{m-1} x \cos^{n+1} x\right) + (m-1)\int \left[\sin^{m-2} x \cdot \cos^n x\right] dx$$

$$(m+n)\int \sin^m x \cos^n x dx = \left(-\sin^{m-1} x \cos^{n+1} x\right) + (m-1)\int \left[\sin^{m-2} x \cdot \cos^n x\right] dx$$

$$\int \sin^m x \cos^n x dx = \frac{-1}{m+n}\left(\sin^{m-1} x \cos^{n+1} x\right) + \frac{m-1}{m+n}\int \left[\sin^{m-2} x \cdot \cos^n x\right] dx$$

==

$$\int \sin^m x \cos^n x dx = \int \sin^m x \left(1 - \sin^2 x\right)^{\frac{n}{2}} dx$$

Proof

$\cos^n x = \left(\cos^2 x\right)^{\frac{n}{2}} = \left(1 - \sin^2 x\right)^{\frac{n}{2}}$

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Reduction Formulas for $Sin^m x\ Cos^n x\ \ (m > n)$

$$\int \sin^m x \cos^n x dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n}\int \sin^m x \cos^{n-2} x dx \quad (m > n)$$

Proof #4:

Prove: $\int \sin^m x \cos^n x dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n}\int \sin^m x \cos^{n-2} x dx \quad (m > n)$

$$\int \sin^m x \cos^n x dx$$

$$= \int \sin^m x \cos^{n-1} x \cos x dx$$

$$= uv - \int v du$$

Let $u = \cos^{n-1} x \sin^m x$

then $du = \left[(n-1)\cos^{n-2} x \cdot -\sin x\right] \cdot \sin^m x + \cos^{n-1} x \cdot \left[m \sin^{m-1} x \cdot (\cos x)\right] dx$

$= \left[-(n-1)\cos^{n-2} x \cdot \sin^{m+1} x + m\cos^n x \cdot \sin^{m-1} x\right] dx$

$dv = \cos x dx \quad$ then $v = \sin x$

$$= \left(\cos^{n-1} x \sin^m x\right) \cdot (\sin x) - \int (\sin x)\left[-(n-1)\cos^{n-2} x \cdot \sin^{n+1} x + m\cos^n x \cdot \sin^{m-1} x\right] dx$$

$$= \left(\sin^{m+1} x \cos^{n-1} x\right) + \int (\sin x)\left[(n-1)\cos^{n-2} x \cdot \sin^{n+1} x - m\cos^n x \cdot \sin^{m-1} x\right] dx$$

$$= \left(\sin^{m+1} x \cos^{n-1} x\right) + \int \left[(n-1)\sin^{m+2} x \cdot \cos^{n-2} x - m\sin^m x \cdot \cos^n x\right] dx$$

$$= \left(\sin^{m+1} x \cos^{n-1} x\right) + (n-1)\int \left[\sin^{m+2} x \cdot \cos^{n-2} x\right] dx - m\int \sin^m x \cos^n x dx$$

$$= \left(\sin^{m+1} x \cos^{n-1} x\right) + (n-1)\int \left[\cos^{n-2} x \cdot \sin^m x \cdot \sin^2 x\right] dx - m\int \sin^m x \cos^n x dx$$

$$= \left(\sin^{m+1} x \cos^{n-1} x\right) + (n-1)\int \left[\cos^{n-2} x \cdot \sin^m x \cdot \left(1 - \cos^2 x\right)\right] dx - m\int \sin^m x \cos^n x dx$$

$$= \left(\sin^{m+1} x \cos^{n-1} x\right) + (n-1)\int \left[\cos^{n-2} x \cdot \sin^m x - \cos^n x \cdot \sin^m x\right] dx - m\int \sin^m x \cos^n x dx$$

$$= \left(\sin^{m+1} x \cos^{n-1} x\right) + (n-1)\int \left[\cos^{n-2} x \cdot \sin^m x\right] dx - (n-1)\int \left[\sin^m x \cdot \cos^n x\right] dx - m\int \sin^m x \cos^n x dx$$

Bring the last two terms to left side

$$\int \sin^m x \cos^n x dx + (n-1)\int \left[\sin^m x \cdot \cos^n x\right] dx + m\int \sin^m x \cos^n x dx = \left(\sin^{m+1} x \cos^{n-1} x\right) + (n-1)\int \left[\cos^{n-2} x \cdot \sin^m x\right] dx$$

$$(m+n)\int \sin^m x \cos^n x dx = \left(\sin^{m+1} x \cos^{n-1} x\right) + (n-1)\int \left[\cos^{n-2} x \cdot \sin^m x\right] dx$$

$$\int \sin^m x \cos^n x dx = \frac{1}{m+n}\left(\sin^{m+1} x \cos^{n-1} x\right) + \frac{n-1}{m+n}\int \left[\cos^{n-2} x \cdot \sin^m x\right] dx$$

$$= \frac{1}{m+n}\left(\sin^{m+1} x \cos^{n-1} x\right) + \frac{n-1}{m+n}\int \left[\sin^m x \cdot \cos^{n-2} x\right] dx$$

Reduction Formulas for Tangent

$$\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$$

Proof #5:

Prove: $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$

$\int \tan^n x dx$

$= \int \tan^{n-2} x \cdot \tan^2 x dx$

$= \int \tan^{n-2} x(\sec^2 x - 1)dx$

$= \int (\tan^{n-2} x \cdot \sec^2 x - \tan^{n-2} x)dx$

$= \int \tan^{n-2} x \cdot \sec^2 x dx - \int \tan^{n-2} x dx$

Let $u = \tan^{n-2} x$ then $du = (n-2)\tan^{n-3} x \cdot \sec^2 x du$

$dv = \sec^2 x dx$ then $v = \tan x$

$uv - \int v du$

$= \left[\tan^{n-2} x(\tan x) - \int (\tan x)(n-2)\tan^{n-3} x \cdot \sec^2 x dx\right] - \int \tan^{n-2} x dx$

$= \tan^{n-1} x - (n-2)\int \tan^{n-2} x \sec^2 x dx - \int \tan^{n-2} x dx$

$\int \tan^n x dx + \int \tan^{n-2} x dx = \tan^{n-1} x - (n-2)\int \tan^{n-2} x \cdot \sec^2 x dx$

$\int (\tan^n x + \tan^{n-2} x)dx = \tan^{n-1} x - (n-2)\int \tan^{n-2} x \cdot \sec^2 x dx$

$\int \tan^{n-2} x(\tan^2 x + 1)dx = \tan^{n-1} x - (n-2)\int \tan^{n-2} x \cdot \sec^2 x dx$

$\int \tan^{n-2} x(\sec^2 x)dx = \tan^{n-1} x - (n-2)\int \tan^{n-2} x \cdot \sec^2 x dx$

$\int \tan^{n-2} x(\sec^2 x)dx + (n-2)\int \tan^{n-2} x \cdot \sec^2 x dx = \tan^{n-1} x$

$(n-1)\int \tan^{n-2} x(\sec^2 x)dx = \tan^{n-1} x$

$\int \tan^{n-2} x(\sec^2 x)dx = \frac{\tan^{n-1} x}{n-1}$

$\int \tan^{n-2} x(1 + \tan^2 x)dx = \frac{\tan^{n-1} x}{n-1}$

$\int \tan^{n-2} x dx + \int \tan^n x dx = \frac{\tan^{n-1} x}{n-1}$

$\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$

Reduction Formulas for Cotangent

$$\int \cot^n x dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x dx$$

Proof #6:

Let $u = \cot^{n-2} x$ then $du = (n-2)\cot^{n-3} x \cdot (-\csc^2 x)du$

$dv = \csc^2 x dx$ then $v = -\cot x$

Prove: $\int \cot^n x dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x dx$

$\int \cot^n x dx$

$= \int \cot^{n-2} x \cdot \cot^2 x dx$

$= \int \cot^{n-2} x(\csc^2 x - 1)dx$

$= \int (\cot^{n-2} x \cdot \csc^2 x - \cot^{n-2} x)dx$

$= \int \cot^{n-2} x \cdot \csc^2 x dx - \int \cot^{n-2} x dx$ ← $uv - \int vdu$

$= [\cot^{n-2} x(-\cot x) - \int (-\cot x)(n-2)\cot^{n-3} x \cdot (-\csc^2 x)dx] - \int \cot^{n-2} x dx$

$= -\cot^{n-1} x - (n-2)\int \cot^{n-2} x \csc^2 x dx - \int \cot^{n-2} x dx$

$\int \cot^n x dx + \int \cot^{n-2} x dx = -\cot^{n-1} x - (n-2)\int \cot^{n-2} x \cdot \csc^2 x dx$

$\int (\cot^n x + \cot^{n-2} x)dx = -\cot^{n-1} x - (n-2)\int \cot^{n-2} x \cdot \csc^2 x dx$

$\int \cot^{n-2} x(\cot^2 x + 1)dx = -\cot^{n-1} x - (n-2)\int \cot^{n-2} x \cdot \csc^2 x dx$

$\int \cot^{n-2} x(\csc^2 x)dx = -\cot^{n-1} x - (n-2)\int \cot^{n-2} x \cdot \csc^2 x dx$

$\int \cot^{n-2} x(\csc^2 x)dx + (n-2)\int \cot^{n-2} x \cdot \csc^2 x dx = -\cot^{n-1} x$

$(n-1)\int \cot^{n-2} x(\csc^2 x)dx = -\cot^{n-1} x$

$\int \cot^{n-2} x(\csc^2 x)dx = \frac{-\cot^{n-1} x}{n-1}$

$\int \cot^{n-2} x(1 + \cot^2 x)dx = \frac{-\cot^{n-1} x}{n-1}$

$\int \cot^{n-2} x dx + \int \cot^n x dx = \frac{-\cot^{n-1} x}{n-1}$

$\int \cot^n x dx = \frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x dx$

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Reduction Formulas for Secant and Cosecant

$$\int \sec^n x dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x dx \qquad \int \csc^n x dx = -\frac{\cot x \csc^{n-2} x}{n-1} + \frac{n-2}{n-1}\int \csc^{n-2} x dx$$

Proof #7:

Prove: $\int \sec^n x dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x dx$

$$\int \sec^n x dx$$
$$= \int \sec^{n-2} x \cdot \sec^2 x dx$$
$$= uv - \int v du$$
$$= \sec^{n-2} x(\tan x) - \int (\tan x)(n-2)\sec^{n-3} x \cdot \sec x \tan x dx$$
$$= \sec^{n-2} x \tan x - (n-2)\int \sec^{n-2} x \tan^2 x dx$$
$$= \sec^{n-2} x \tan x - (n-2)\int \sec^{n-2} x(\sec^2 x - 1)dx$$
$$= \sec^{n-2} x \tan x - (n-2)\int (\sec^n x - \sec^{n-2} x)dx$$
$$= \sec^{n-2} x \tan x - (n-2)\int \sec^n x dx + (n-2)\int \sec^{n-2} x dx$$

Let $u = \sec^{n-2} x$ then $du = (n-2)\sec^{n-3} x \cdot \sec x \tan x dx$

$dv = \sec^2 x dx$ then $v = \tan x$

Bring the first integral to left side

$$\int \sec^n x dx + (n-2)\int \sec^n x dx = \sec^{n-2} x \tan x + (n-2)\int \sec^{n-2} x dx$$
$$(n-1)\int \sec^n x dx = \sec^{n-2} x \tan x + (n-2)\int \sec^{n-2} x dx$$
$$\int \sec^n x dx = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x dx$$

Proof #8:

Prove: $\int \csc^n x dx = -\frac{\cot x \csc^{n-2} x}{n-1} + \frac{n-2}{n-1}\int \csc^{n-2} x dx$

$$\int \csc^n x dx$$
$$= \int \csc^{n-2} x \cdot \csc^2 x dx$$
$$= uv - \int v du$$
$$= \csc^{n-2} x(-\cot x) - \int (-\cot x)(n-2)\csc^{n-3} x \cdot (-\csc x \cot x)dx$$
$$= -\csc^{n-2} x \cot x - (n-2)\int \csc^{n-2} x \cot^2 x dx$$
$$= -\csc^{n-2} x \cot x - (n-2)\int \csc^{n-2} x(\csc^2 x - 1)dx$$
$$= -\csc^{n-2} x \cot x - (n-2)\int (\csc^n x - \csc^{n-2} x)dx$$
$$= -\csc^{n-2} x \cot x - (n-2)\int \csc^n x dx + (n-2)\int \csc^{n-2} x dx$$

Let $u = \csc^{n-2} x$ then $du = (n-2)\csc^{n-3} x \cdot -\csc x \cot x dx$

$dv = \csc^2 x dx$ then $v = -\cot x$

Bring the first integral to left side

$$\int \csc^n x dx + (n-2)\int \csc^n x dx = -\csc^{n-2} x \cot x + (n-2)\int \csc^{n-2} x dx$$
$$(n-1)\int \csc^n x dx = -\csc^{n-2} x \cot x + (n-2)\int \csc^{n-2} x dx$$
$$\int \csc^n x dx = \frac{-1}{n-1}\csc^{n-2} x \cot x + \frac{n-2}{n-1}\int \csc^{n-2} x dx$$

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Reduction Formulas for *Sin(mx)Cos(nx)*

$$\int \sin mx \cos nx dx = -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)} + C \quad (m \neq \pm n)$$

Proof #9:

Prove: $\int \sin mx \cos nx dx = -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)} + C \quad (m \neq \pm n)$

Recall: $\sin mx \cos nx = \frac{1}{2}\sin(m-n)x + \frac{1}{2}\sin(m+n)x$

$$\int \sin mx \cos nx dx = \int \left[\frac{1}{2}\sin(m-n)x + \frac{1}{2}\sin(m+n)x\right]dx$$

$$= \frac{1}{2}\int [\sin(m-n)x + \sin(m+n)x]dx$$

$$= \frac{1}{2}\left[\frac{-\cos(m-n)x}{m-n} + \frac{-\cos(m+n)x}{m+n}\right] + C$$

$$= \frac{-\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)} + C$$

Note: $\sin mx \cos nx = \frac{1}{2}\sin(m-n)x + \frac{1}{2}\sin(m+n)x$

Proof

$$LS = \frac{1}{2}\sin(m-n)x + \frac{1}{2}\sin(m+n)x$$

$$= \frac{1}{2}\sin(mx-nx) + \frac{1}{2}\sin(mx+nx)$$

$$= \frac{1}{2}(\sin mx \cos nx - \cos mx \sin nx) + \frac{1}{2}(\sin mx \cos nx + \cos mx \sin nx)$$

$$= \frac{1}{2}\sin mx \cos nx - \frac{1}{2}\cos mx \sin nx + \frac{1}{2}\sin mx \cos nx + \frac{1}{2}\cos mx \sin nx$$

$$= \sin mx \cos nx$$

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Reduction Formulas for *Sin(mx)Sin(nx)*

$$\int \sin mx \sin nx dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + C \quad (m \neq \pm n)$$

Proof #10:

Prove: $\int \sin mx \sin nx dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + C \quad (m \neq \pm n)$

Recall: $\sin mx \sin nx = \frac{1}{2}\cos(m-n)x - \frac{1}{2}\cos(m+n)x$

$$\int \sin mx \sin nx dx = \int \left[\frac{1}{2}\cos(m-n)x - \frac{1}{2}\cos(m+n)x \right] dx$$

$$= \frac{1}{2}\int [\cos(m-n)x - \cos(m+n)x] dx$$

$$= \frac{1}{2}\left[\frac{\sin(m-n)x}{m-n} - \frac{\sin(m+n)x}{m+n} \right] + C$$

$$= \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + C$$

Note: $\sin mx \sin nx = \frac{1}{2}\cos(m-n)x - \frac{1}{2}\cos(m+n)x$

Proof

$$LS = \frac{1}{2}\cos(m-n)x - \frac{1}{2}\cos(m+n)x$$

$$= \frac{1}{2}\cos(mx-nx) - \frac{1}{2}\cos(mx+nx)$$

$$= \frac{1}{2}(\cos mx \cos nx + \sin mx \sin nx) - \frac{1}{2}(\cos mx \cos nx - \sin mx \sin nx)$$

$$= \frac{1}{2}\cos mx \cos nx + \frac{1}{2}\sin mx \sin nx - \frac{1}{2}\cos mx \cos nx + \frac{1}{2}\sin mx \sin nx$$

$$= \sin mx \sin nx$$

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Reduction Formulas for *Cos(mx)Cos(nx)*

$$\int \cos mx \cos nx dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C \quad (m \neq \pm n)$$

Proof #11:

Prove: $\int \cos mx \cos nx dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C \quad (m \neq \pm n)$

Recall: $\cos mx \cos nx = \frac{1}{2}\cos(m-n)x + \frac{1}{2}\cos(m+n)x$

$$\int \cos mx \cos nx dx = \int \left[\frac{1}{2}\cos(m-n)x + \frac{1}{2}\cos(m+n)x\right]dx$$

$$= \frac{1}{2}\int [\cos(m-n)x + \cos(m+n)x]dx$$

$$= \frac{1}{2}\left[\frac{\sin(m-n)x}{m-n} + \frac{\sin(m+n)x}{m+n}\right] + C$$

$$= \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C$$

Note: $\cos mx \cos nx = \frac{1}{2}\cos(m-n)x + \frac{1}{2}\cos(m+n)x$

Proof

$$LS = \frac{1}{2}\cos(m-n)x + \frac{1}{2}\cos(m+n)x$$

$$= \frac{1}{2}\cos(mx-nx) + \frac{1}{2}\cos(mx+nx)$$

$$= \frac{1}{2}(\cos mx \cos nx + \sin mx \sin nx) + \frac{1}{2}(\cos mx \cos nx - \sin mx \sin nx)$$

$$= \frac{1}{2}\cos mx \cos nx + \frac{1}{2}\sin mx \sin nx + \frac{1}{2}\cos mx \cos nx - \frac{1}{2}\sin mx \sin nx$$

$$= \cos mx \cos nx$$

Proving some Reduction Formulas

Date

Table of Trigonometric Integrals

$$\int \sin^2 x dx = \frac{x}{2} - \frac{\sin 2x}{4} + C \text{ or } = \frac{x}{2} - \frac{1}{2}\sin x \cos x + C$$

$$\int \cos^2 x dx = \frac{x}{2} + \frac{\sin 2x}{4} + C \text{ or } = \frac{x}{2} + \frac{1}{2}\sin x \cos x + C$$

$$\int \sin^n x dx = \frac{-1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x dx$$

$$\int \cos^n x dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x dx$$

$$\int \sin^m x \cos^n x dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n}\int \sin^m x \cos^{n-2} x dx \quad (m > n)$$

$$\int \sin^m x \cos^n x dx = -\frac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2} x \cos^n x dx \quad (m < n)$$

$$\int \tan x dx = \ln|\sec x| + C = -\ln|\cos x| + C$$

$$\int \tan^m x dx = \frac{\tan^{m-1} x}{m-1} - \int \tan^{m-2} x dx$$

$$\int \cot x dx = -\ln|\csc x| + C = \ln|\sin x| + C$$

$$\int \cot^m x dx = -\frac{\cot^{m-1} x}{m-1} - \int \cot^{m-2} x dx$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

$$\int \sec^m x dx = \frac{\tan x \sec^{m-2} x}{m-1} + \frac{m-2}{m-1}\int \sec^{m-2} x dx$$

$$\int \csc x dx = \ln|\csc x - \cot x| + C$$

$$\int \csc^m x dx = -\frac{\cot x \csc^{m-2} x}{m-1} + \frac{m-2}{m-1}\int \csc^{m-2} x dx$$

$$\int \sin mx \sin nx dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + C \quad (m \neq \pm n)$$

$$\int \sin mx \cos nx dx = -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)} + C \quad (m \neq \pm n)$$

$$\int \cos mx \cos nx dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C \quad (m \neq \pm n)$$