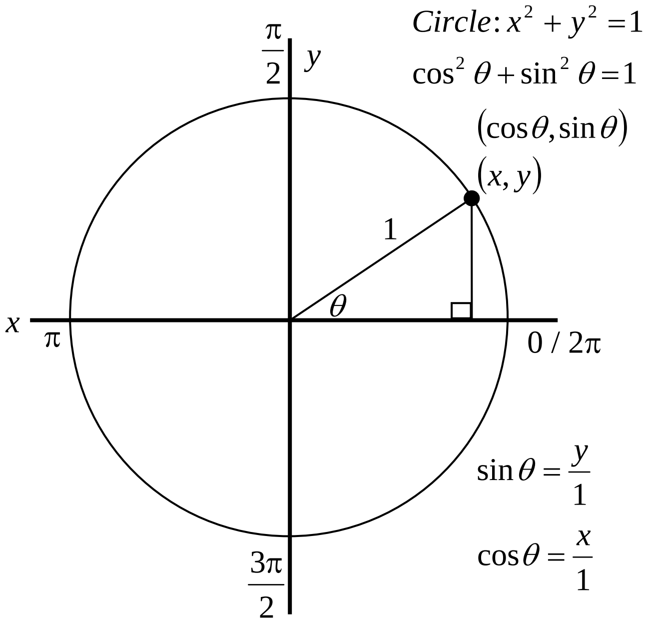
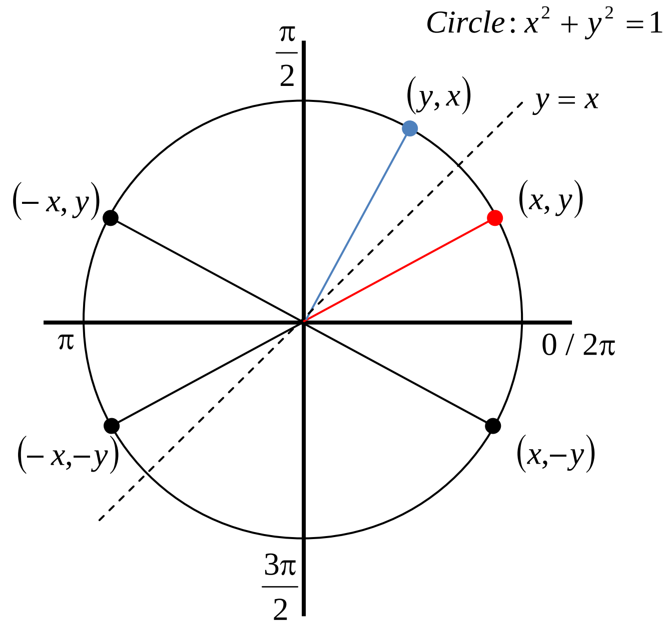
Proving the Formulas by Graphs & Algebras

Date:

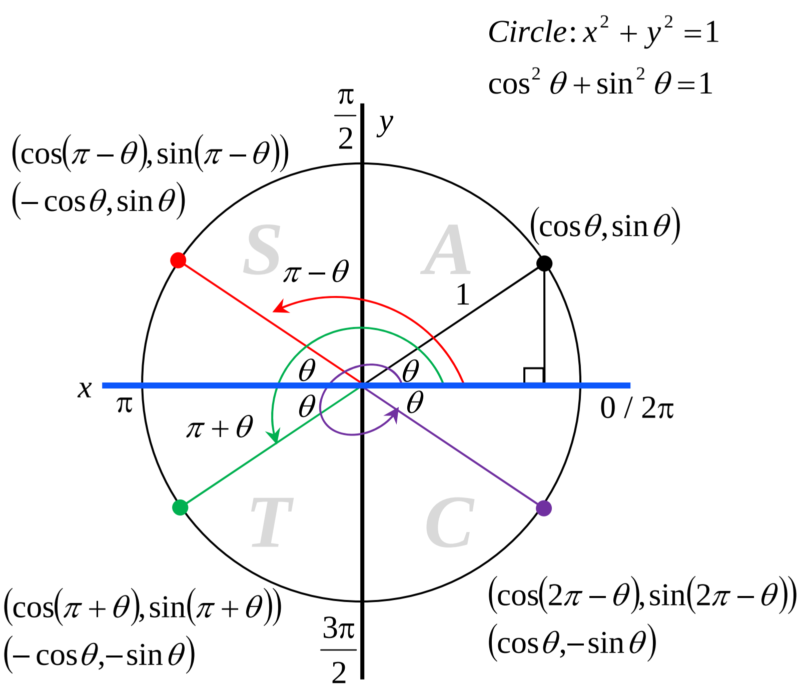
Circle Coordinates in terms of Trigonometric Angles



Reflection Concepts in Cartesian Plane



Related Acute Formulas



Related Acute Angle Formulas				
sin	$(\pi-\theta)$	***sin***	θ	***S***
cos		***- cos***		
tan		***- tan***		
sin	$(\pi+\theta)$	***- sin***	θ	***T***
cos		***- cos***		
tan		***tan***		
sin	$(2\pi-\theta)$	***- sin***	θ	***C***
cos		***cos***		
tan		***- tan***		
sin	$(-\theta)$	***- sin***	θ	***C***
cos		***cos***		
tan		***- tan***		

Proving the Formulas by Graphs & Algebras

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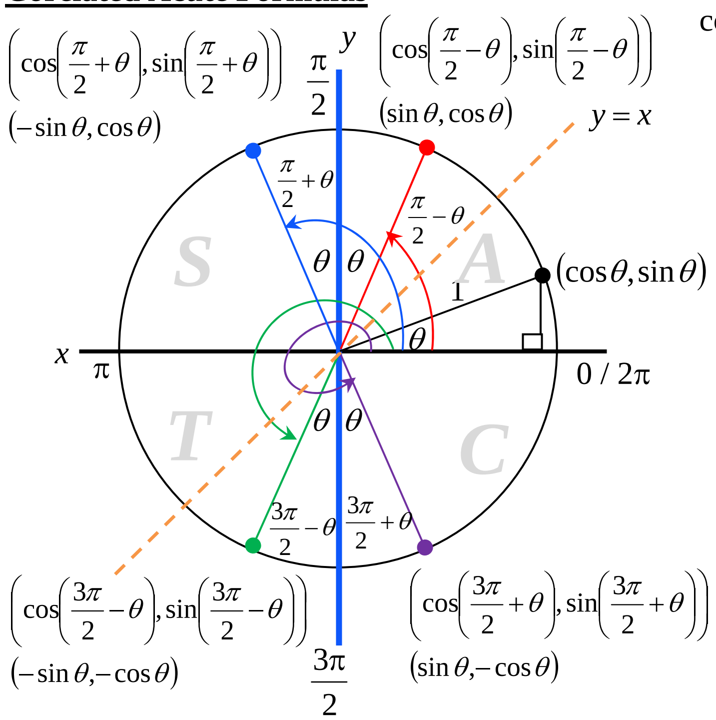
Corelated Acute Formulas



$Circle: x^2 + y^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$

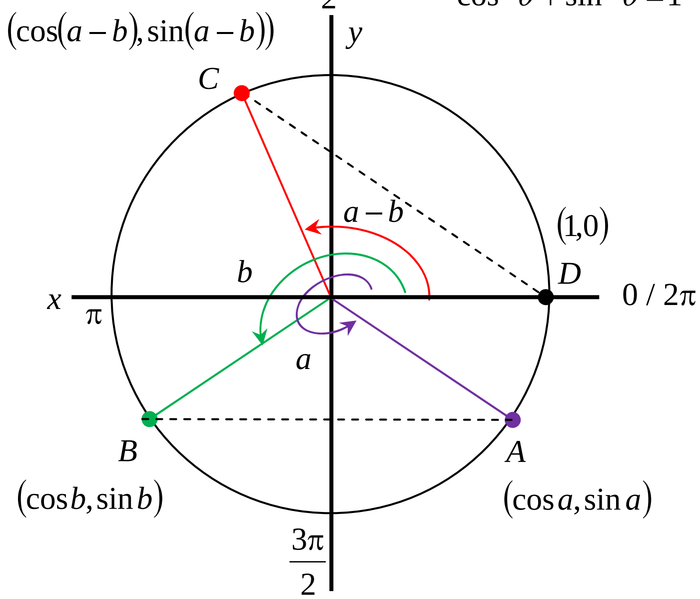
Corelated Acute Angle Formulas

sin	$\left(\frac{\pi}{2}-\theta\right)$	**cos**	θ	**A**
cos		**sin**		
tan		**cot**		
sin	$\left(\frac{\pi}{2}+\theta\right)$	**cos**	θ	**S**
cos		**- sin**		
tan		**- cot**		
sin	$\left(\frac{3\pi}{2}-\theta\right)$	**- cos**	θ	**T**
cos		**- sin**		
tan		**cot**		
sin	$\left(\frac{3\pi}{2}+\theta\right)$	**- cos**	θ	**C**
cos		**sin**		
tan		**- cot**		

Addition and Subtraction Formulas

$Circle: x^2 + y^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$



$AB = CD$ (Constructed)

$$\sqrt{(\cos a - \cos b)^2 + (\sin a - \sin b)^2} = \sqrt{[\cos(a-b) - 1]^2 + [\sin(a-b) - 0]^2}$$

$LS = \cos^2 a - 2\cos a\cos b + \cos^2 b + \sin^2 a - 2\sin a\sin b$

$= 1 + 1 - 2\cos a\cos b - 2\sin a\sin b$

$= 2 - 2\cos a\cos b - 2\sin a\sin b$

$RS = \cos^2(a-b) - 2\cos(a-b) + 1 + \sin^2(a-b)$

$= 1 - 2\cos(a-b) + 1$

$= 2 - 2\cos(a-b)$

Let LS = RS

$2 - 2\cos a\cos b - 2\sin a\sin b = 2 - 2\cos(a-b)$

$2\cos a\cos b + 2\sin a\sin b = 2\cos(a-b)$

$\therefore \cos a\cos b + \sin a\sin b = \cos(a-b)$

$\because \cos(a+b) = \cos[a - (-b)]$

$\therefore \cos(a+b) = \cos a\cos(-b) + \sin a\sin(-b)$

$= \cos a\cos b + \sin a(-\sin b)$

$= \cos a\cos b - \sin a\sin b$

Proving the Formulas by Graphs & Algebras

Date:

$$\because \sin(a+b)=\cos\left[\frac{\pi}{2}-(a+b)\right]=\cos\left[\left(\frac{\pi}{2}-a\right)-b\right]$$
$$\therefore \sin(a+b)=\cos\left(\frac{\pi}{2}-a\right)\cos b+\sin\left(\frac{\pi}{2}-a\right)\sin b$$
$$=\sin a\cos b+\cos a\sin b$$

$$\because \sin(a-b)=\sin[a+(-b)]$$
$$\therefore \sin(a-b)=\sin a\cos(-b)+\cos a\sin(-b)$$
$$=\sin a\cos b-\cos a\sin b$$

$$\tan(a+b)=\frac{\sin(a+b)}{\cos(a+b)}$$
$$=\frac{\sin a\cos b+\cos a\sin b}{\cos a\cos b-\sin a\sin b}$$
(Divide every term by $\cos a\cos b$)
$$=\frac{\frac{\sin a\cos b}{\cos a\cos b}+\frac{\cos a\sin b}{\cos a\cos b}}{\frac{\cos a\cos b}{\cos a\cos b}-\frac{\sin a\sin b}{\cos a\cos b}}$$
$$=\frac{\frac{\sin a}{\cos a}+\frac{\sin b}{\cos b}}{1-\frac{\sin a}{\cos a}\cdot\frac{\sin b}{\cos b}}$$
$$=\frac{\tan a+\tan b}{1-\tan a\tan b}$$

$$\because \tan(a-b)=\tan[a+(-b)]$$
$$\therefore \tan(a-b)=\frac{\tan a+\tan(-b)}{1-\tan a\tan(-b)}$$
$$=\frac{\tan a-\tan b}{1+\tan a\tan b}$$

Addition and Subtraction Formulas

$$\sin(x+y)=\sin x\cos y+\cos x\sin y$$
$$\sin(x-y)=\sin x\cos y-\cos x\sin y$$
$$\cos(x+y)=\cos x\cos y-\sin x\sin y$$
$$\cos(x-y)=\cos x\cos y+\sin x\sin y$$
$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}$$
$$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}$$

Double Angle Formulas

$\sin 2x=2\sin x\cos x$

Proof

$\sin 2x=\sin(x+x)$

$=\sin x\cos x+\cos x\sin x$

$=2\sin x\cos x$

$\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$

Proof

$\tan 2x=\tan(x+x)$

$=\dfrac{\tan x+\tan x}{1-\tan x\tan x}$

$=\dfrac{2\tan x}{1-\tan^2 x}$

$\cos 2x=\cos^2 x-\sin^2 x$

$\cos 2x=2\cos^2 x-1$

$\cos 2x=1-2\sin^2 x$

Proof

$\cos 2x=\cos(x+x)$

$=\cos x\cos x-\sin x\sin x$

$=\cos^2 x-\sin^2 x$

or $=\cos^2 x-(1-\cos^2 x)$

$=2\cos^2 x-1$

or $=(1-\sin^2 x)-\sin^2 x$

$=1-2\sin^2 x$

Double Angle Formulas

1) $\sin 2x=2\sin x\cos x$
2) $\cos 2x=\cos^2 x-\sin^2 x$
   $=2\cos^2 x-1$
   $=1-2\sin^2 x$
3) $\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$

Half Angle Formulas (Optional)

$\sin\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{2}}$	$\cos\frac{x}{2}=\pm\sqrt{\frac{1+\cos x}{2}}$	$\tan\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{1+\cos x}}=\frac{1-\cos x}{\sin x}$ *or* $\frac{\sin x}{1+\cos x}$
Proof $\cos 2x=1-2\sin^2 x$ $2\sin^2 x=1-\cos 2x$ $\sin^2 x=\frac{1-\cos 2x}{2}$ $\sin x=\pm\sqrt{\frac{1-\cos 2x}{2}}$ Replace x with $\frac{x}{2}$ $\sin\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{2}}$	Proof $\cos 2x=2\cos^2 x-1$ $2\cos^2 x=1+\cos 2x$ $\cos^2 x=\frac{1+\cos 2x}{2}$ $\cos x=\pm\sqrt{\frac{1+\cos 2x}{2}}$ Replace x with $\frac{x}{2}$ $\cos\frac{x}{2}=\pm\sqrt{\frac{1+\cos x}{2}}$	Proof $\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$ $\tan\frac{x}{2}=\frac{\pm\sqrt{\frac{1-\cos x}{2}}}{\pm\sqrt{\frac{1+\cos x}{2}}}=\pm\sqrt{\frac{\frac{1-\cos x}{2}}{\frac{1+\cos x}{2}}}$ $=\pm\sqrt{\frac{1-\cos x}{1+\cos x}}=\frac{\sqrt{1-\cos x}}{\sqrt{1+\cos}}\cdot\frac{\sqrt{1-\cos x}}{\sqrt{1-\cos x}}$ $=\frac{1-\cos x}{\sqrt{1-\cos^2 x}}=\frac{1-\cos x}{\sqrt{\sin^2 x}}=\frac{1-\cos x}{\sin x}$

Product to Sum Formulas (Optional)

$\sin a\sin b=\frac{\cos(a-b)-\cos(a+b)}{2}$	$\cos a\cos b=\frac{\cos(a+b)+\cos(a-b)}{2}$	$\tan a\tan b=\frac{\cos(a-b)-\cos(a+b)}{\cos(a+b)+\cos(a-b)}$
Proof : $\cos(a+b)=\cos a\cos b-\sin a\sin b \quad (1)$ $\cos(a-b)=\cos a\cos b+\sin a\sin b \quad (2)$ $(2)-(1)$ $\cos(a-b)-\cos(a+b)=2\sin a\sin b$ $\therefore \sin a\sin b=\frac{\cos(a-b)-\cos(a+b)}{2}$	Proof : $\cos(a+b)=\cos a\cos b-\sin a\sin b \quad (1)$ $\cos(a-b)=\cos a\cos b+\sin a\sin b \quad (2)$ $(1)+(2)$ $\cos(a+b)+\cos(a-b)=2\cos a\cos b$ $\therefore \cos a\cos b=\frac{\cos(a+b)+\cos(a-b)}{2}$	Proof : $\tan a\tan b=\frac{\sin a\sin b}{\cos a\cos b}$ $=\frac{\cos(a-b)-\cos(a+b)}{\cos(a+b)+\cos(a-b)}$

$\sin a\cos b=\frac{\sin(a+b)+\sin(a-b)}{2}$	$\cos a\sin b=\frac{\sin(a+b)-\sin(a-b)}{2}$
Proof : $\sin(a+b)=\sin a\cos b+\cos a\sin b \quad (1)$ $\sin(a-b)=\sin a\cos b-\cos a\sin b \quad (2)$ $(1)+(2)$ $\sin(a+b)+\sin(a-b)=2\sin a\cos b$ $\therefore \sin a\cos b=\frac{\sin(a+b)+\sin(a-b)}{2}$	Proof : $\sin(a+b)=\sin a\cos b+\cos a\sin b \quad (1)$ $\sin(a-b)=\sin a\cos b-\cos a\sin b \quad (2)$ $(1)-(2)$ $\sin(a+b)-\sin(a-b)=2\cos a\sin b$ $\therefore \cos a\sin b=\frac{\sin(a+b)-\sin(a-b)}{2}$

Sum to Product Formulas (Optional)

$$\sin a+\sin b=2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

Proof :

$$\text{Recall}:\sin a\cos b=\frac{\sin(a+b)+\sin(a-b)}{2}$$

$$\text{Let } x=\frac{a+b}{2}\quad y=\frac{a-b}{2}$$

$$x+y=\frac{a+b}{2}+\frac{a-b}{2}=\frac{2a}{2}=a$$

$$x-y=\frac{a+b}{2}-\frac{a-b}{2}=\frac{2b}{2}=b$$

$$\sin x\cos y\to\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$=\frac{\sin(x+y)+\sin(x-y)}{2}=\frac{\sin a+\sin b}{2}$$

$$\sin a+\sin b=2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\sin a-\sin b=2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

Proof :

$$\text{Recall}:\cos a\sin b=\frac{\sin(a+b)-\sin(a-b)}{2}$$

$$\text{Let } x=\frac{a+b}{2}\quad y=\frac{a-b}{2}$$

$$x+y=\frac{a+b}{2}+\frac{a-b}{2}=\frac{2a}{2}=a$$

$$x-y=\frac{a+b}{2}-\frac{a-b}{2}=\frac{2b}{2}=b$$

$$\cos x\sin y\to\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$=\frac{\sin(x+y)-\sin(x-y)}{2}=\frac{\sin a-\sin b}{2}$$

$$\sin a-\sin b=2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\cos a+\cos b=2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

Proof :

$$\text{Recall}:\cos a\cos b=\frac{\cos(a-b)+\cos(a+b)}{2}$$

$$\text{Let } x=\frac{a+b}{2}\quad y=\frac{a-b}{2}$$

$$x+y=\frac{a+b}{2}+\frac{a-b}{2}=\frac{2a}{2}=a$$

$$x-y=\frac{a+b}{2}-\frac{a-b}{2}=\frac{2b}{2}=b$$

$$\cos x\cos y\to\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$=\frac{\cos(x-y)+\cos(x+y)}{2}=\frac{\cos b+\cos a}{2}$$

$$\cos a+\cos b=2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\cos a-\cos b=-2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

Proof :

$$\text{Recall}:\sin a\sin b=\frac{\cos(a-b)-\cos(a+b)}{2}$$

$$\text{Let } x=\frac{a+b}{2}\quad y=\frac{a-b}{2}$$

$$x+y=\frac{a+b}{2}+\frac{a-b}{2}=\frac{2a}{2}=a$$

$$x-y=\frac{a+b}{2}-\frac{a-b}{2}=\frac{2b}{2}=b$$

$$\sin x\sin y\to\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$=\frac{\cos(x-y)-\cos(x+y)}{2}=\frac{\cos b-\cos a}{2}$$

$$\cos b-\cos a=2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\cos a-\cos b=-2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

Triple Angle Formulas (Optional)

$\sin 3x = 3\sin x - 4\sin^3 x$	$\cos 3x = 4\cos^3 x - 3\cos x$
Proof : $\sin(2x+x) = \sin 2x\cos x + \cos 2x\sin x$ $= (2\sin x\cos x)\cos x + (1-2\sin^2 x)(\sin x)$ $= 2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x$ $= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$ $= 3\sin x - 4\sin^3 x$	Proof : $\cos(2x+x) = \cos 2x\cos x - \sin 2x\sin x$ $= (2\cos^2 x - 1)\cos x - (2\sin x\cos x)(\sin x)$ $= 2\cos^3 x - \cos x - 2\sin^2 x\cos x$ $= 2\cos^3 x - \cos x - 2(1-\cos^2 x)\cos x$ $= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$ $= 4\cos^3 x - 3\cos x$

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$$

Proof:

$$\tan(2x+x) = \frac{\tan 2x + \tan x}{1-\tan 2x\tan x}$$

$$= \frac{\frac{2\tan x}{1-\tan^2 x} + \tan x}{1-\frac{2\tan x}{1-\tan^2 x}\tan x}$$

$$= \frac{\frac{2\tan x + \tan x - \tan^3 x}{1-\tan^2 x}}{\frac{1-\tan^2 x - 2\tan^2 x}{1-\tan^2 x}}$$

$$= \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$$