Inverse Functions (Misconceptions) Date:

Definition of Inverse Functions

An inverse function is a function that undoes another function: If an input *x* into the function *f* produces an output *y*, then putting *y* into the inverse function *g* produces the output *x*, and vice versa. i.e., $f(x)=y$, and $g(y)=x$

Misconception 1:

Calling the of inverse of $y=f(x)$ is $y=f^{-1}(x)$

Let *x* be the independent variable, and let *y* be the dependent variable of a function *f*. Then for $y=f(x)$.

$f^{-1}(f(x))=x$

$f^{-1}(y)=x$ since $y=f(x)$

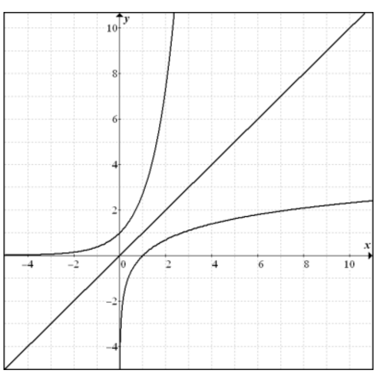
Example 1: Misconception 1 about Inverse Functions

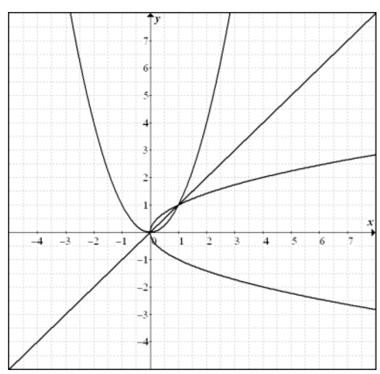
Given $f(x)=1.8x+32$, find f^{-1}

Proper way to do this:

Misconception 2:

If the horizontal axis represents the independent variable and the vertical axis represents the dependent variable, the graph of *f* and f^{-1} may be drawn on the same axes. The resulting graphs are symmetric about the line $y=x$





Fact: $y=\ln x$ and $y=e^{x}$ are inversed related to each other.

$y=x^{2}$ and $y^{2}=x$ are inversed related to each other.

<u>Example 2: Misconception 2: Calculating Wages</u>

If a student reveals that she earns $10 per hour at a retail store, if the student worked h hours, he can earned m where $m = 10h$.

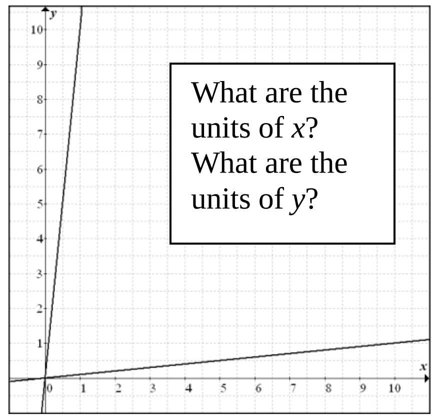
How many hours he will need to work to earn $300?

$m = 10h$	Independent variable: h Dependent variable: m
$\frac{m}{10} = h$	Independent variable: m Dependent variable: h

How many hours he needs to work to earn m?

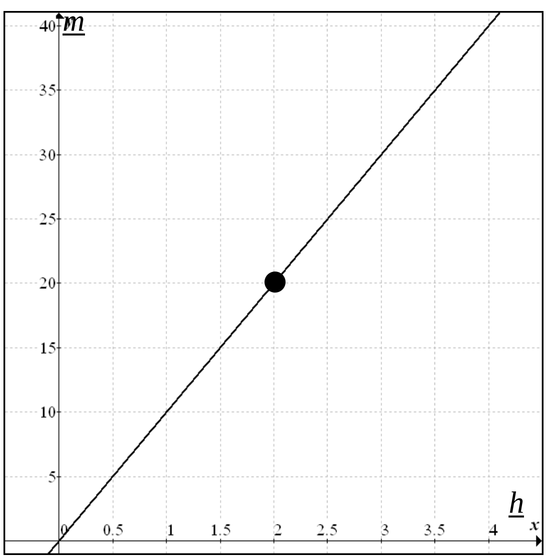
Hours worked ➔ f ➔ money earned	$f(h) = m$
Hours worked 🡐 f^{-1} 🡐money earned	$f^{-1}(m) = h$

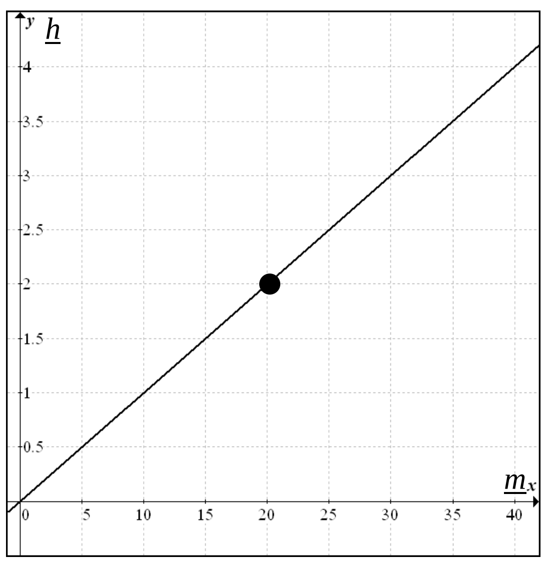
The importance of graphing a function and its inverse on separate axes is critical when working with mathematical models. For example, the function $m = f(h) = 10h$ gives the amount of money earned by a person who works h hours at $10 an hour. The inverse function, $h = f^{-1}(m) = \frac{m}{10}$, gives the number of hours an employee must work at $10 an hour in order to earn m dollars.



Although the equation $y = 10x$ and $y = 0.1x$ may be graph simultaneously on the same set of axes, the physical interpretation of the context being modeled is lost.

In contrast, when the function and its inverse are graphed on separate axes, the meaning of the variables and underlying real-world context is clear.





The meaning of coordinates (20, 2) and (2, 20) depends on the meaning of the variables assigned to each axis.

Example 3:

Determine the inverse f^{-1} if $f(x) = (2x+8)^3$

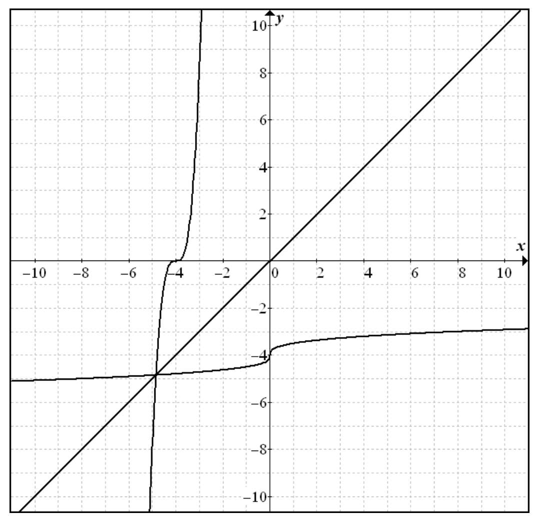
Composition of a function and its inverse

Steps of changing f to f^{-1} (Method 1)

(1) Isolate x respects to y
(2) Denote x by $f^{-1}(y)$
(3) Note that $(f^{-1} \circ f)(x) = (f \circ f^{-1})(x) = x$.

Steps of changing f to f^{-1} (Method 2)

(1) Change x to y, and y to x.
(2) Subject y so changed, and denote it by $f^{-1}(x)$
(3) Note that $(f^{-1} \circ f)(x) = (f \circ f^{-1})(x) = x$.



Example 4

Given $h(x) = 2x - 3$, determine

a) $h^{-1}(x)$

b) $(h \circ h^{-1})(x)$

c) $(h^{-1} \circ h)(x)$.

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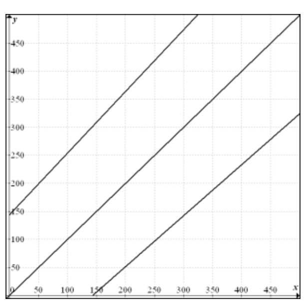
Example 5

Let $y = F(x) = \dfrac{5}{x-2}, x \neq 2$. Find the formula for $F^{-1}(y)$.

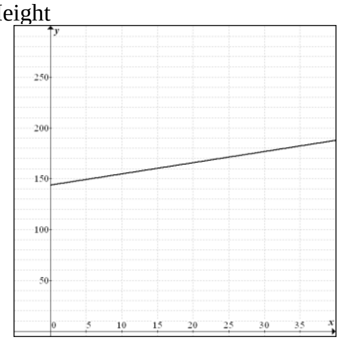
Example 6

The relationship between height, $h(x)$, in cm and footprint length, x, in cm is given by $h(x) = 1.1x + 143.6$. Use this relationship to predict the footprint length for a person who is 170 cm tall.

A person who is 170 cm tall may have a footprint length of 24 cm.

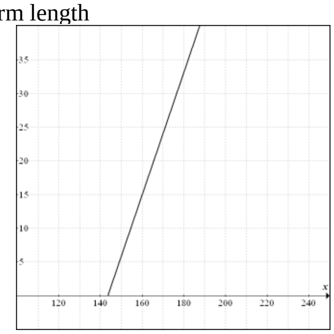


Height



Arm length

Arm length



Height

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1) Given $f(x)=2x+5$, find an expression for each function.

a) $f^{-1}(x)$ b) $f \circ f^{-1}(x)$ c) $f^{-1} \circ f(x)$

2) Given $f(x)=\dfrac{x-1}{x+1}$, find an expression for each function.

a) $f^{-1}(x)$ b) $f \circ f^{-1}(x)$ c) $f^{-1} \circ f(x)$

Answers

1a) $\dfrac{x-5}{2}$ b) x c) x 1a) $\dfrac{1+x}{1-x}$ b) x c) x