

Composition of Functions by Algebra

Date: _____

Given two functions f and g , the **composite function** $f \circ g$ is defined by $(f \circ g)(x) = f(g(x))$

Example 1

If $f(x) = \sqrt{x}$ and $g(x) = x + 5$, find each of the following:

- a) $f(g(4))$ b) $g(f(4))$ c) $f(g(x))$ d) $g(f(x))$

Example 2

Let $f(x) = 2x + 3$ and $g(x) = \frac{5x+1}{2x-3}$, find

- a) $(g \circ f)(x)$ b) $(f \circ g)(x)$.

Example 3

Let $h(x) = 3x + 12$ and $k(x) = \sqrt{x}$. What are the domain and range of $k \circ h$?

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Example 4

We are given that $h(x) = f(g(x)) = \frac{1}{1+x^2}$.

(a) If $f(x) = \frac{1}{x}$, what is $g(x)$?

b) If $g(x) = x^2$, what is $f(x)$?

Example 5

If $f(x) = \sqrt{2-x^2}$ and $f(g(x)) = \sqrt{2-x^6}$, find $g(x)$

Example 6

Use the functions $f(x) = 3x+1$, $g(x) = x^3$, $h(x) = \frac{1}{x+1}$ and $u(x) = \sqrt{x}$ to find expressions for the indicated composite function.

a) $h \circ (f \circ u)$

b) $(f \circ g) \circ u$

c) $g \circ (h \circ u)$

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Example 7

Let $f(x) = x + 4$ and $g(x) = (x - 2)^2$. Find a function u so that $f(g(u(x))) = 4x^2 - 8x + 8$.

Exercise

- 1) Given $f(x) = 2x + 1$ and $g(x) = 3x + 1$ find
 - a) $f(3)$
 - b) $g(f(3))$
 - c) $g(3)$
 - d) $f(g(3))$

- 2) Find $f(g(x))$ and $g(f(x))$ for each pair of functions.
 - a) $f(x) = 3x + 4$; $g(x) = -2x + 5$
 - b) $f(x) = x^2 + 5x$; $g(x) = 2x + 1$
 - c) $f(x) = 2x^2 - 3x + 1$; $g(x) = 7 - 4x$

- 3) Given $f(x) = 3x^2 - 1$, find $f(g(x))$ and $g(f(x))$ for each function $g(x)$.
 - a) $g(x) = x + 2$
 - b) $g(x) = 1 - 2x$
 - c) $g(x) = x^2$
 - d) $g(x) = x^2 + x$
 - e) $g(x) = 2x^2 - 3x$
 - f) $g(x) = \frac{1}{x}, x \neq 0$

- 4) For each pair of functions, find $f \circ g(x)$, $g \circ f(x)$, $f \circ f(x)$, and $g \circ g(x)$
 - a) $f(x) = \sqrt{x}$; $g(x) = 4 - 2x$
 - b) $f(x) = \sqrt{2x}$; $g(x) = 1 + 3x$
 - c) $f(x) = \frac{x}{x+1}$; $g(x) = x^2 - 1$
 - d) $f(x) = 2^x$; $g(x) = 3x - 4$

- 5) Find two functions whose composite function is $k(x)$.
 - a) $k(x) = x^6 + 2x^3 + 1$
 - b) $k(x) = (x - 4)^2 + 3(x - 4) + 4$
 - c) $k(x) = \sqrt{3x - 2}$
 - d) $k(x) = \frac{1}{x + 3}$

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6) Given $f(x) = 2x + 1$

Find a) $f(f(x))$ b) $f \circ f \circ f(x)$ c) $f \circ f \circ f \circ f(x)$

7) From the functions listed in the box, find two whose composite function is $h(x)$.

a) $h(x) = (x+1)^2$

b) $h(x) = \sqrt{x-3}$

c) $h(x) = x^2 - 6x + 9$

d) $h(x) = x - 2$

$$e(x) = x - 3$$

$$f(x) = x^2$$

$$g(x) = \sqrt{x}$$

$$k(x) = x + 1$$

8) Given $f(x) = 2x + 5$, find an expression for each function.

a) $f^{-1}(x)$

b) $f \circ f^{-1}(x)$

c) $f^{-1} \circ f(x)$

9) Given $f(x) = \frac{x-1}{x+1}$, find an expression for each function.

a) $f^{-1}(x)$

b) $f \circ f^{-1}(x)$

c) $f^{-1} \circ f(x)$

Answers

1a) 7 b) 22 c) 10 d) 21 2a) $-6x + 19; -6x - 3$ b) $4x^2 + 14x + 6; 2x^2 + 10x + 1$

c) $32x^2 - 100x + 78; -8x^2 + 12x + 3$ 3a) $3x^2 + 12x + 11; 3x^2 + 1$ b) $12x^2 - 12x + 2; 3 - 6x^2$

c) $3x^4 - 1; 9x^4 - 6x^2 + 1$ d) $3x^4 + 6x^3 + 3x^2 - 1; 9x^4 - 3x^2$ e) $12x^4 - 36x^3 + 27x^2 - 1; 18x^4 - 21x^2 + 5$

f) $\frac{3}{x^2} - 1; \frac{1}{3x^2 - 1}$ 4a) $\sqrt{4-2x}; 4-2\sqrt{x}; \sqrt[4]{x}; 4x-4$ b) $\sqrt{2+6x}; 1+3\sqrt{2x}; \sqrt{2\sqrt{2x}}; 9x+4$

c) $\frac{x^2-1}{x^2}; \frac{x^2}{(x+1)^2} - 1; \frac{x}{2x+1}; x^4 - 2x^2$ d) $2^{3x-4}; 3(2^x) - 4; 2^{2^x}; 9x - 6$

5a) Answers may vary, for example. a) $f(x) = x^2; g(x) = x^3 + 1; f(g(x))$

b) $f(x) = x^2 + 3x + 4; g(x) = x - 4; f(g(x))$ c) $f(x) = \sqrt{x}; g(x) = 3x - 2; f(g(x))$

d) $f(x) = \frac{1}{x}; g(x) = x + 3; f(g(x))$ 6a) $4x + 3$ b) $8x + 7$ c) $16x + 15$ 7a) $f(k(x))$ b) $g(e(x))$

c) $f(e(x))$ d) $k(e(x))$