# Composition of Functions

Date:

**Definition of a Composite Function**

If $g: A \rightarrow B$ and $f: B \rightarrow C$, then the composite function of *f* and *g* is the function

$$f \circ g : A \rightarrow C$$

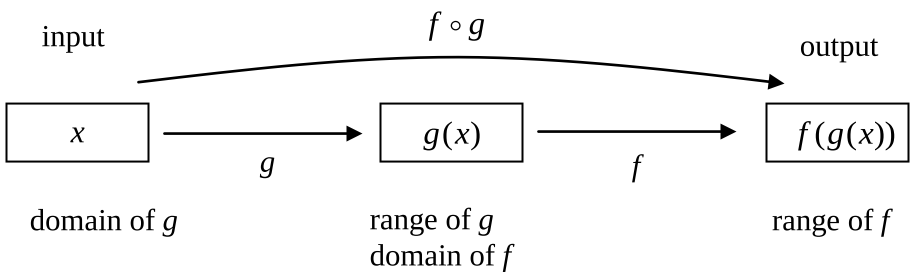
defined by

$$(f \circ g)(x) = f(g(x)).$$

**Note:**

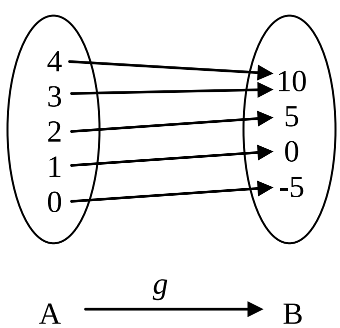
$A$ = domain of *g* $B$ = range of *g* & domain of *f* $C$ = range of *f*.

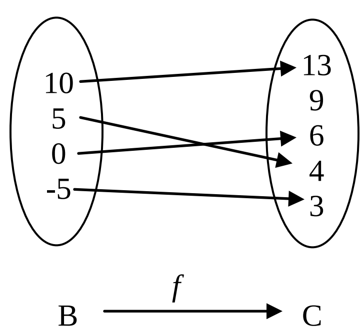
The best way to think of composition is in terms of arrow diagrams.




**Example 1**

If *f* and *g* are given by the following arrow diagrams, draw an arrow diagram for $f \circ g$.



**The Composition of a Function and its Inverse**

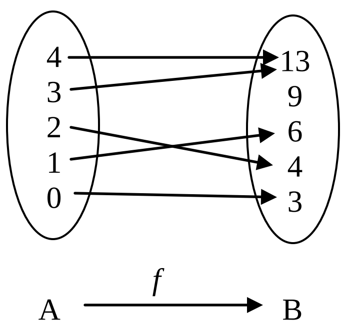
If both $f$ and $f^{-1}$ are functions, then

- $(f^{-1} \circ f)(x) = x$ for all $x$ in the domain of $f$, and
- $(f \circ f^{-1})(x) = x$ for all $x$ in the domain of $f^{-1}$.

**Example 2**

$f$ is given by the following arrow diagram. Write in ordered pairs for

(a) $f$ (b) $f^{-1}$ (c) $f^{-1} \circ f$ (d) $f \circ f^{-1}$.




Composition of discrete "relations"

**Example 3**

The arrow diagrams below represent relations $k$ and $k^{-1}$. Draw an arrow diagram for $k^{-1} \circ k$. Explain why $(k^{-1} \circ k)(x) = x$ is not true for all x in the domain of $k$. What condition would guarantee that, in general, $(f^{-1} \circ f)(x) = x$ for all x in the domain of $f$?