

Combining Two Functions: Products and Quotients

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When two function $f(x)$ and $g(x)$, are combined to form the function $(f \times g)(x)$, the new function is called the **product** of f and g . For any given value of x , the function value is represented by $f(x) \times g(x)$. The graph of $f \times g$ can be obtained from the graphs of functions f and g by multiplying each y-coordinate of f by the corresponding y-coordinate from g .

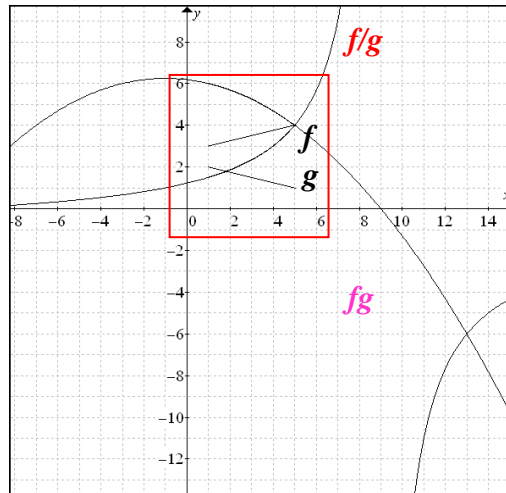
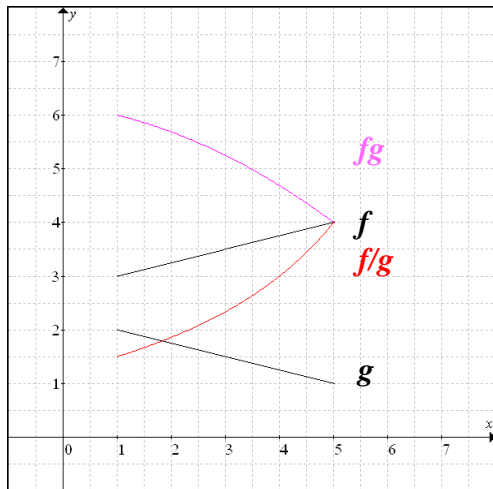
Algebraically, $f \times g$ is defined as $(f \times g)(x) = f(x) \cdot g(x)$

The domain of $f \times g$ is the intersection of the domains of f and g .

Similar concepts apply to the **quotient** of functions. $\frac{f}{g}$
 \Rightarrow Restrictions!

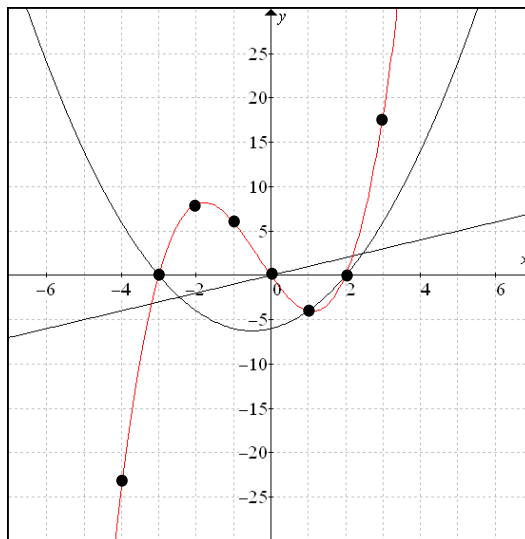
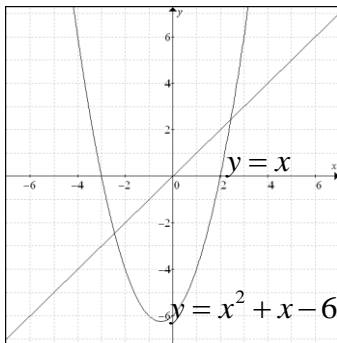
Example 1: Constructing the Product and Quotient of two functions graphically

Given the graphs of f and g , sketch the graphs of i) $f \times g$ ii) $\frac{f}{g}$



Example 2: Constructing the Product of two functions graphically

Determine the graph domain and range of $y = (f \times g)(x)$, given the graphs of $f(x) = x^2 + x - 6$ and $g(x) = x$.



x	$f(x)$	$g(x)$	$(f \times g)(x)$
-4	6	-4	-24
-3	0	-3	0
-2	-4	-2	8
-1	-6	-1	6
0	-6	0	0
1	-4	1	-4
2	0	2	0
3	6	3	18
4	14	4	56

$$\begin{aligned}
 y &= fg \\
 &= (x^2 + x - 6)(x) \\
 &= x^3 + x^2 - 6x \\
 &= (x)(x + 3)(x - 2)
 \end{aligned}$$

The domain of the product function is the intersection of the domains of f and g , $\{x \in R\}$. Range is $\{y \in R\}$

Example 3: Constructing the product of two functions algebraically

Let $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{2}x - 2$.

a) Find the equation of the functions i) $f \times g$ ii) $\frac{f}{g}(x)$.

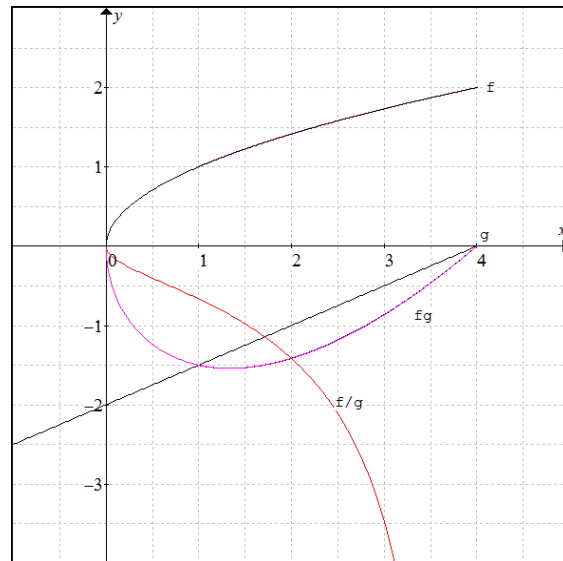
b) Sketch the combined functions in interval $[-1, 4]$.

$$fg = \sqrt{x}(0.5x - 2)$$

$$\frac{f}{g} = \frac{\sqrt{x}}{0.5x - 2}$$

$$= \frac{2\sqrt{x}}{x - 4}$$

x	f(x)	g(x)	fg	f/g
-1	/	-2.5	/	/
0	0	-2	0	0
1	1	-1.5	-1.5	-0.7
2	1.4	-1	-1.4	-1.4
3	1.7	-0.5	-0.85	-3.4
4	2	0	0	/



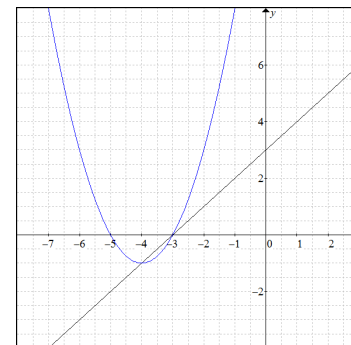
Example 4: Product and Quotient

Let $f(x) = x + 3$ and $g(x) = x^2 + 8x + 15$. Determine the equation for each combined function. Sketch a

graph of the combined functions i) $f \times g$ ii) $\frac{f}{g}$ and state its domain and range.

a) $y = f(x)g(x)$
 $= (x + 3)(x^2 + 8x + 15)$
 $= x^3 + 11x^2 + 39x + 45$
 $= (x + 3)(x + 3)(x + 5)$
 $= (x + 3)^2(x + 5)$

b) $y = f(x) / g(x)$
 $= \frac{x + 3}{x^2 + 8x + 15} = \frac{x + 3}{(x + 3)(x + 5)}$
 $= \frac{1}{x + 5}$

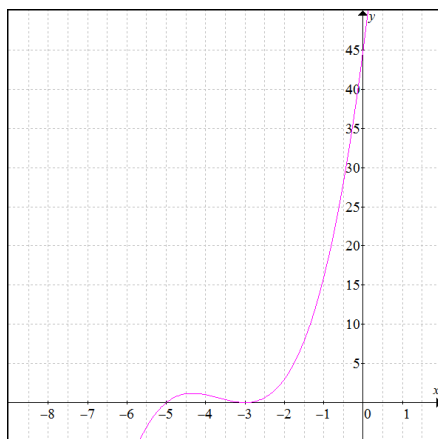


x-int: -3 and -5

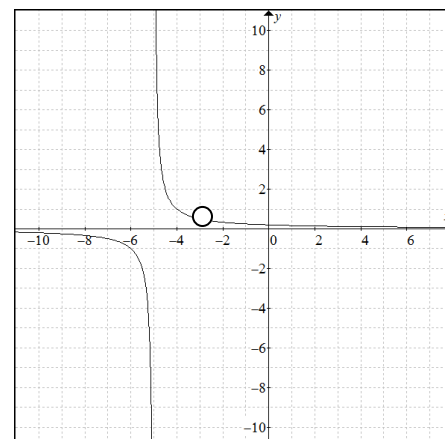
y-int: 45

D: $x \in R$

R: $y \in R$



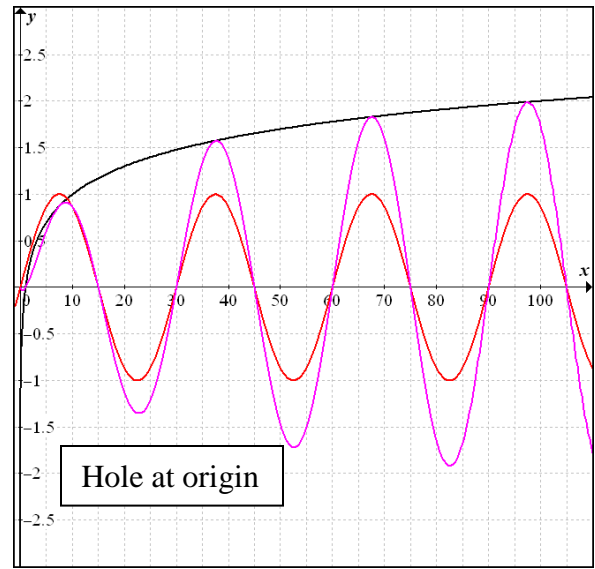
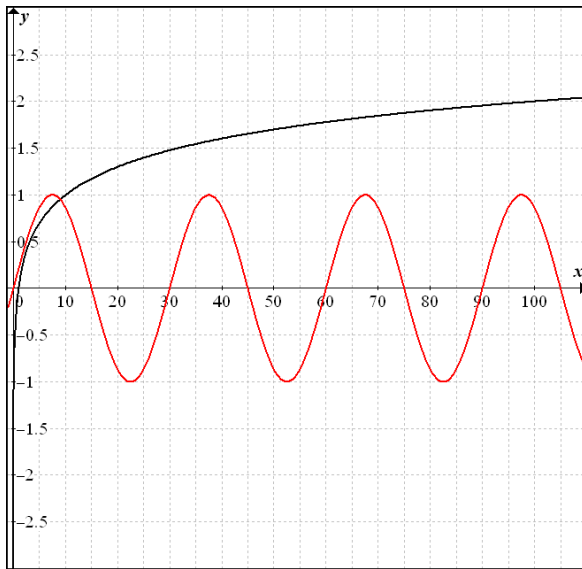
VA: $x = -5$
 Hole at $x = -3$
 $(-3, 0.5)$
 D: $x \neq -3, -5$
 R: $y \neq 0, 0.5$



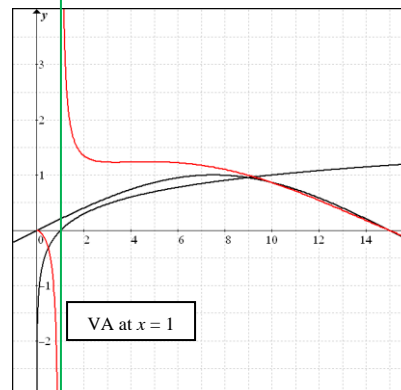
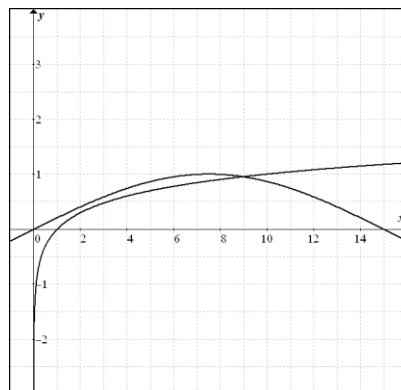
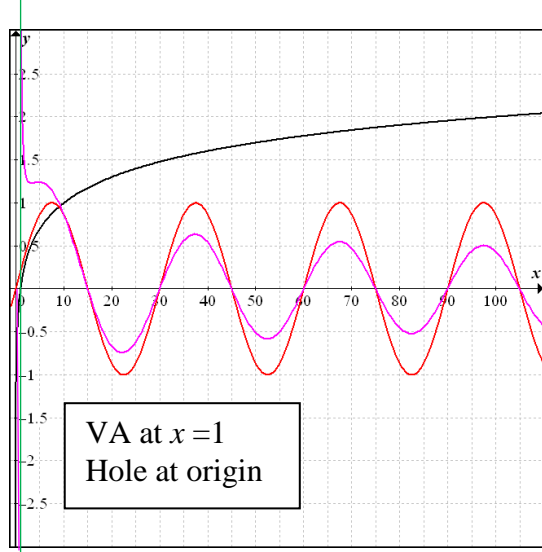
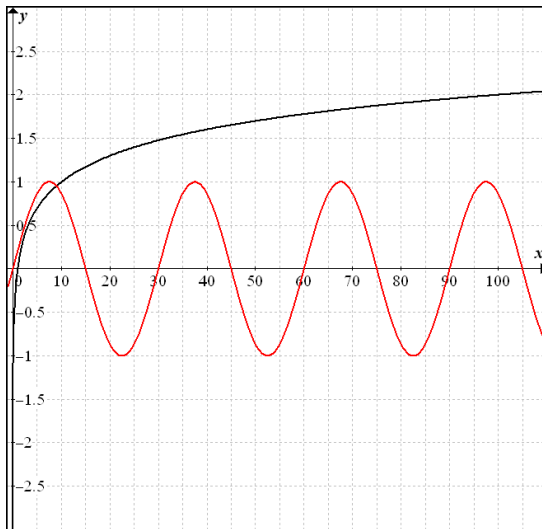
Example 5: Product and Quotient: (Different type of functions)

For $f(x) = \sin \frac{\pi}{15} x$, $g(x) = \log_{10} x$,

a) Create an image of $(f \times g)(x)$,



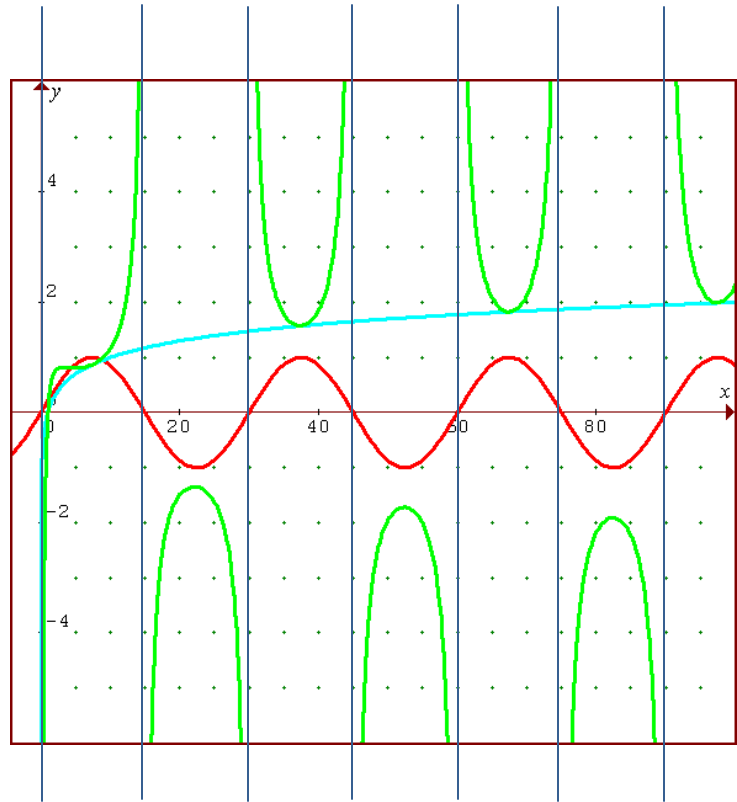
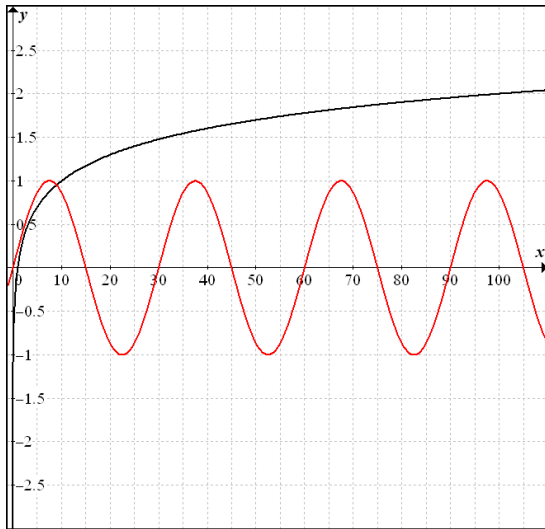
b) Create an image of $\left(\frac{f}{g}\right)(x)$,



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c) Create an image of $\left(\frac{g}{f}\right)(x)$,



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Exercise1) Determine $(f \times g)(x)$ for each of the following pairs of functions.

a) $f(x) = x - 7$, $g(x) = x + 7$

b) $f(x) = \sqrt{x+10}$, $g(x) = \sqrt{x+10}$

c) $f(x) = 7x^2$, $g(x) = x - 9$

d) $f(x) = -4x - 7$, $g(x) = 4x + 7$

e) $f(x) = 2\sin x$, $g(x) = \frac{1}{x-1}$

f) $f(x) = \log(x+4)$, $g(x) = 2^x$

2) For each of the following pairs of functions, state the domain of $(f \times g)(x)$.

a) $f(x) = \frac{1}{x^2 - 5x - 14}$, $g(x) = \sec x$

b) $f(x) = 99^x$, $g(x) = \log(x-8)$

c) $f(x) = \sqrt{x+81}$, $g(x) = \csc x$

d) $f(x) = \log(x^2 + 6x + 9)$, $g(x) = \sqrt{x^2 - 1}$

3) If $f(x) = x^2 - 25$,

a) Determine the equation of the product function $f(x) \times \frac{1}{f(x)}$

b) Determine the domain and sketch the graph of the product function in part a).

4) Determine two functions, f and g , whose product would result in each of the following functions.

a) $(f \times g)(x) = 4x^2 - 81$

b) $(f \times g)(x) = 8\sin^3 x + 27$

c) $(f \times g)(x) = 4x^{\frac{5}{2}} - 3x^{\frac{3}{2}} + x^{\frac{1}{2}}$

d) $(f \times g)(x) = \frac{6x-5}{2x+1}$

5) Let $f(x) = x - 2$ and $g(x) = x^2 - 4$. Develop an algebraic and a graphical model for each combined function. Then, give the domain and range of the combined function. Identify any holes or asymptotes.

a) $y = f(x)g(x)$

b) $y = \frac{f(x)}{g(x)}$

c) $y = \frac{g(x)}{f(x)}$

6) Let $f(x) = \sqrt{25 - x^2}$ and $g(x) = \sin x$.a) Graph $f(x)$ and describe its shape. Is this function even, odd, or neither?b) Graph $g(x)$ on the same set of axes. Is this function even, odd, or neither?c) Predict the shape of $y = f(x)g(x)$. Sketch a graph of your prediction.d) Give the domain of $y = f(x)g(x)$. Estimate the range. To two decimal places.

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Answers

1a) $x^2 - 49$ b) $x + 10$ c) $7x^3 - 63x^2$ d) $-16x^2 - 56x - 49$ e) $\frac{2 \sin x}{x-1}$ f) $2^x \log(x+4)$

2a) $x \neq 7, -2, \frac{\pi}{2} + n\pi, n \in I$ b) $x > 8$ c) $x \geq -81, x \neq n\pi, n \in I$ d) $x < -3, -3 < x \leq -1$ or $x \geq 1$

3a) $y = 1$ b) $x \neq \pm 5, x \in R$

4) Answers vary a) $f(x) = 2x - 9; g(x) = 2x + 9$ b) $f(x) = 2 \sin x + 3; g(x) = 4 \sin^2 x - 6 \sin x + 9$

c) $f(x) = \sqrt{x}; g(x) = 4x^2 - 3x + 1$ d) $f(x) = 6x - 5; g(x) = \frac{1}{2x+1}$

5a) $y = x^3 - 2x^2 - 4x + 8, x \& y \in R$ b) $y = \frac{1}{x+2}, x \neq \pm 2, y \neq \frac{1}{4} \& 0$ Hole: $\left(2, \frac{1}{4}\right),$

asymptotes: $x = -2, y = 0$ c) $y = x + 2, x \neq 2, y \neq 4, \text{ hole: } (2, 4)$ 6a) semi-circle, even b) odd

d) $\{-5 \leq x \leq 5\} \& \{-4.76 \leq y \leq 4.76\}$