Combining Two Functions: Sums & Differences

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When two functions $f(x)$ and $g(x)$ are combined to form the function $(f+g)(x)$, the new function is called the sum of *f* and *g*. For any given value of *x*, the value of the function is represented by $f(x)+g(x)$. The graph of $f+g$ can be obtained from the graphs of functions *f* and *g* by adding corresponding y-coordinates.

Similarly, the difference of two functions, $f-g$, is $(f-g)(x)=f(x)-g(x)$. The graph $f-g$ can be obtained by subtracting the y-coordinate of *g* from the y-coordinate of *f* for every pair of corresponding x-value.

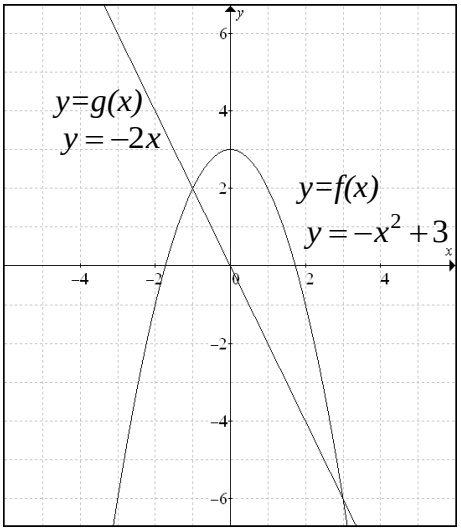
The domain of $y=f(x)\pm g(x)$ consists of all values of x which belong to the domain of $f(x)$ and to the domain of $g(x)$.

Example 1: Selecting a strategy to combine functions by addition and subtraction

Given $f(x)=-x^2+3$ and $g(x)=-2x$, determine the graphs of $f(x)+g(x)$ and $f(x)-g(x)$.

Method 1: By Completing the Table (Grade 9 method)

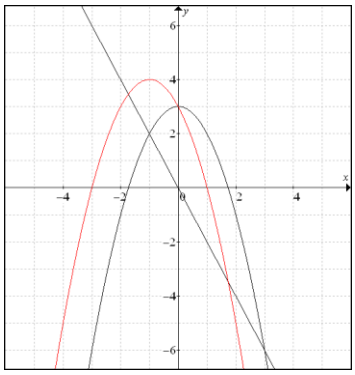
x	$f(x)$	$g(x)$	$f(x)+g(x)$	$f(x)-g(x)$
-3	-6	6	0	-12
-2	-1	4	3	-5
-1	2	2	4	0
-0.5	2.75	1	3.75	1.75
0	3	0	3	3
1	2	-2	0	4
2	-1	-4	-5	3
3	-6	-6	-12	0



Method 2: By Algebra

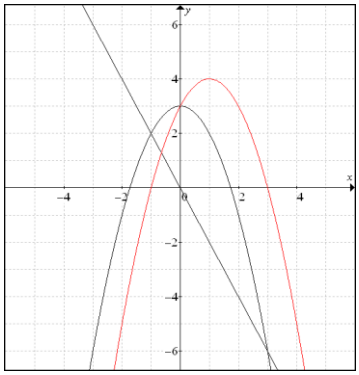
$f(x)=-x^2+3$ and $g(x)=-2x$

$(f+g)(x)=f(x)+g(x)$
$=(-x^2+3)+(-2x)$
$=-x^2-2x+3$
$----------$
$=-(x^2+2x)+3$
$=-(x^2+2x+1-1)+3$
$=-(x^2+2x+1)+1+3$
$=-(x+1)^2+4$



$y=(f+g)(x)$
$y=-(x+1)^2+4$

$(f-g)(x)=f(x)-g(x)$
$=(-x^2+3)-(-2x)$
$=-x^2+2x+3$
$----------$
$=-(x^2-2x)+3$
$=-(x^2-2x+1-1)+3$
$=-(x^2-2x+1)+1+3$
$=-(x-1)^2+4$



$y=(f-g)(x)$
$y=-(x-1)^2+4$

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PS: Not all combined functions can be drawn.

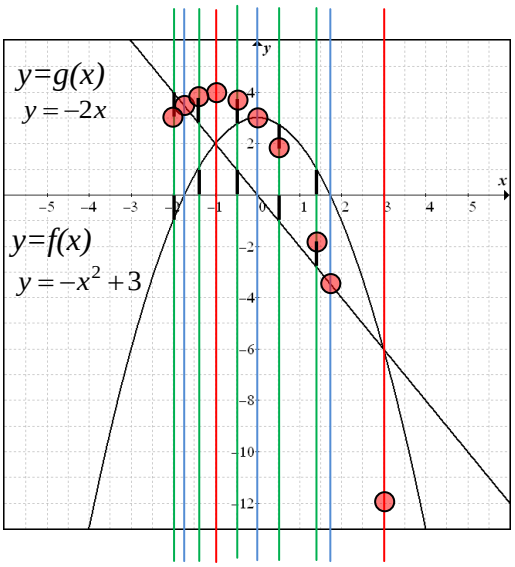
If $f(x)\&\,g(x)$ are same type of functions, most likely the properties can lead to the diagram, else if $f(x)\&\,g(x)$ are different type of functions, then method 3 below becomes an option.

Method 3: By mapping the key points for *y = f(x) + g(x)*

OPTIONAL for NOW!! Comeback after Example 2

- Select some key points in the diagram,

  (Intercepts, intersections, *y = +1*, *y = -1* or other of your choice)
- Draw a vertical line,
- Perform the operation to combine the function.



Example 2:

a) Graph the function $f(x)=x+2$ and $g(x)=x^2$.

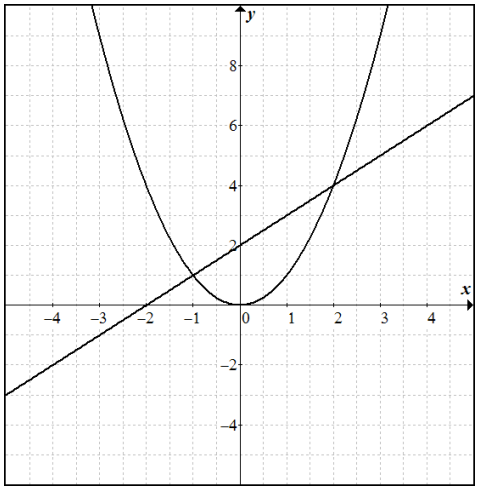
b) Write $y=f(x)-g(x)$ as a function of x.

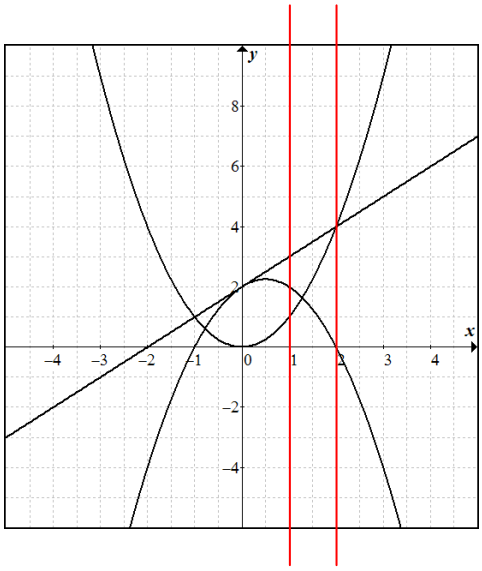
c) Use the result of part a) to graph the function $y=x+2-x^2$.

d) What is the domain of $y=x+2-x^2$?

x	-3	-2	-1	0	1	2	3
f(x)	-1	0	1	2	3	4	5
g(x)	9	4	1	0	1	4	9
f(x) - g(x)	-10	-4	0	2	2	0	-4

Since $f(x)=x+2$ and $g(x)=x^2$ are defined for all real values of x, the domain of $y=x+2-x^2$ is the set of all real numbers.





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Example 3:

a) Graph the function $f(x) = x$ and $g(x) = \frac{1}{x}$, $x \neq 0$.

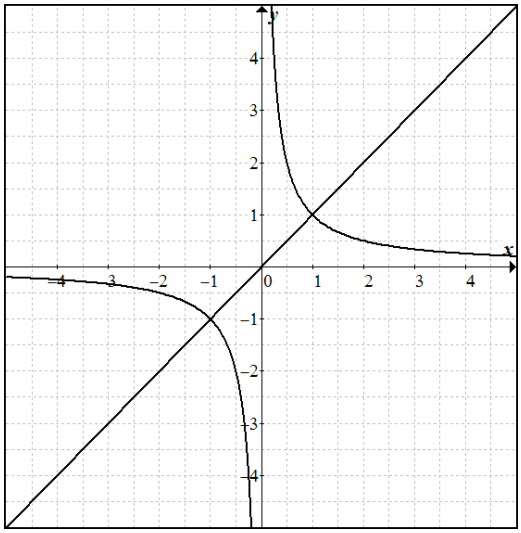
b) Write $y = f(x) + g(x)$ as a function of x.

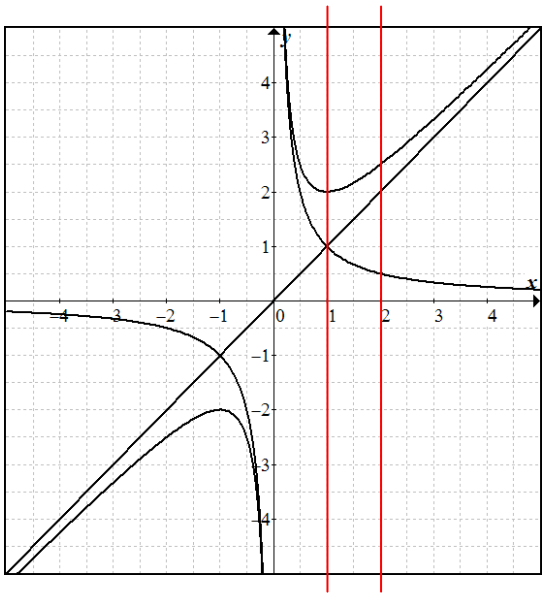
c) Use the result of part a) to graph the function $y = x + \frac{1}{x}$, $x \neq 0$.

d) What is the domain of $y = x + \frac{1}{x}$?

x	-4	-2	-1	-0.5	0	0.5	1	2	4
$f(x)$	-4	-2	-1	-0.5	0	0.5	1	2	4
$g(x)$	-0.25	-0.5	-1	-2	/	2	1	0.5	0.25
$f(x) + g(x)$	-4.25	-2.5	-2	-2.5	/	2.5	2	2.5	4.25

$$y = x + \frac{1}{x} = \frac{x^2 + 1}{x}, \quad x \neq 0$$





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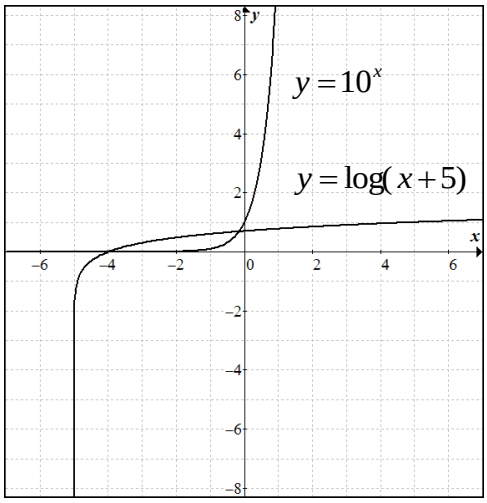
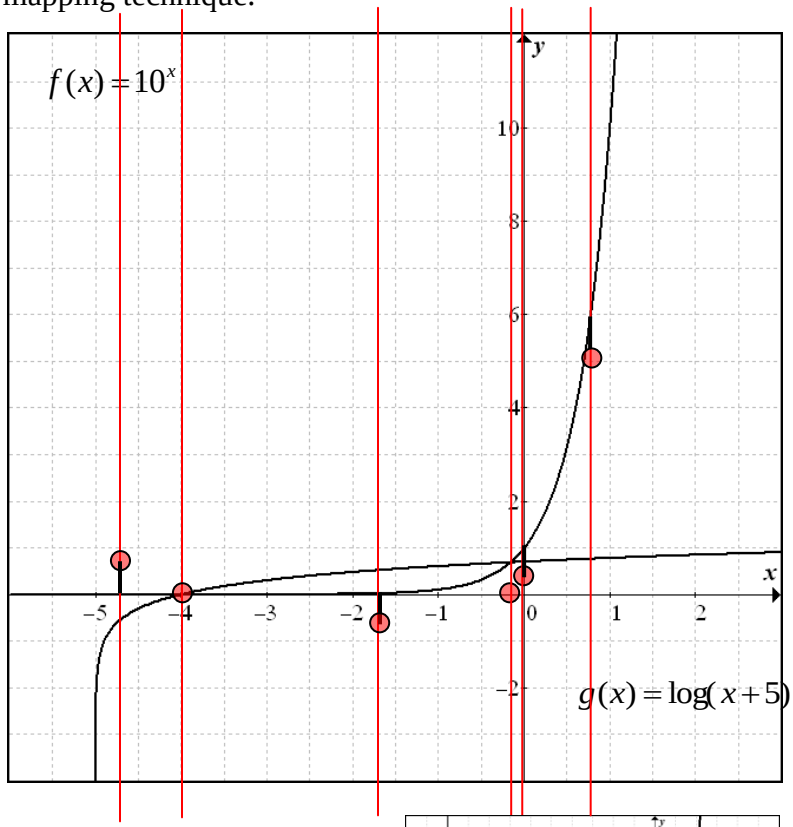
Example 4: Connecting the domains of the sum and difference of two functions

Determine the domain and range of $(f-g)(x)$ and $(f+g)(x)$ if $f(x)=10^x$ and $g(x)=\log(x+5)$.

Provide an image for $(f-g)(x)$ using the mapping technique.





$f(x)=10^x,\quad x\in R$

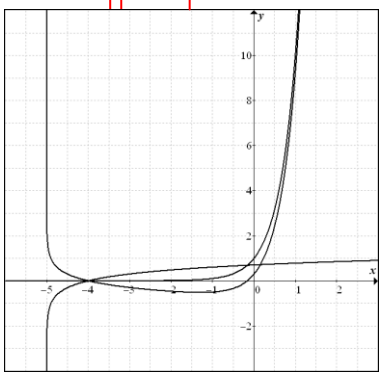
$g(x)=\log(x+5),\quad x>-5$

$(f-g)(x)=f(x)-g(x)$

$=10^x-\log(x+5)$

$(f+g)(x)=f(x)+g(x)$

$=10^x+\log(x+5)$



The domain of the functions $(f-g)(x)$ and $(f+g)(x)$ is $\{x\in R\mid x>-5\}$

The intersection of two sets of numbers that contains all numbers that are common to both sets.
$\therefore x>-5$

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Operations on Functions

Let the domain of f be A and the domain of g be B

Addition	$(f+g)(x)=f(x)+g(x)$	Domain = $A\cap B$
Subtraction	$(f-g)(x)=f(x)-g(x)$	Domain = $A\cap B$
Multiplication	$(fg)(x)=f(x)g(x)$	Domain = $A\cap B$
Division	$(\frac{f}{g})(x)=\frac{f(x)}{g(x)}$	Domain = $A\cap B, g(x)\neq 0$

Example 5:

If $f(x)=x^2-5x-1$, where $x\in A=\{x\,|\,-4\le x\le 1\}$, and $g(x)=2x+3$, where $x\in B=\{x\,|\,-2\le x\le 5\}$, find the functions and its domain of a) $f+g$, b) $f-g$, c) fg, and d) $\frac{f}{g}$.

a) $(f+g)(x)=f(x)+g(x)$

$=(x^2-5x-1)+(2x+3)$

$=x^2-3x+2$

Domain $=A\cap B$

$=\{x\,|\,-2\le x\le 1\}$

b) $(f-g)(x)=f(x)-g(x)$

$=(x^2-5x-1)-(2x+3)$

$=x^2-7x-4$

Domain $=A\cap B$

$=\{x\,|\,-2\le x\le 1\}$

c) $(fg)(x)=f(x)g(x)$

$=(x^2-5x-1)(2x+3)$

$=2x^3-7x^2-17x-3$

Domain $=A\cap B$

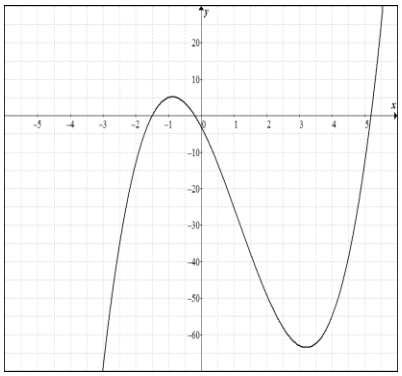
$=\{x\,|\,-2\le x\le 1\}$

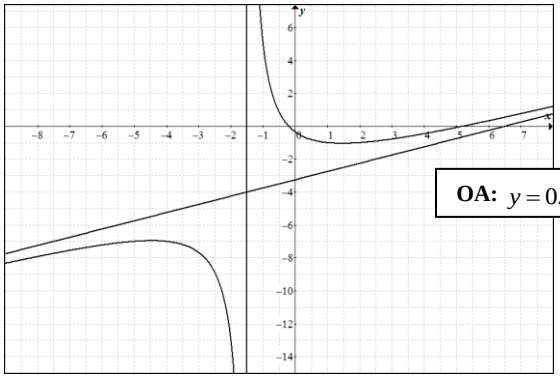
d) $(\frac{f}{g})(x)=\frac{f(x)}{g(x)}$

$=\frac{(x^2-5x-1)}{2x+3}$

Domain $=A\cap B$

$=\{x\,|\,-2\le x\le 1, x\neq -1.5\}$





OA: $y=0.5x-3.25$

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Exercise:

1) Let f = {(-4, 4), (-2, 4), (1, 3), (3, 5), (4, 6)} and g = {(-4, 2), (-2, 1), (0, 2), (1, 2), (2, 2), (4, 4)}. Determine.

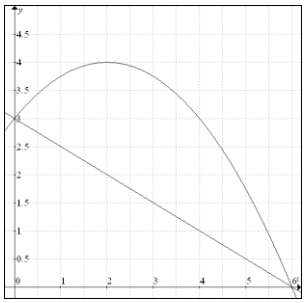
a) $f + g$ b) $g + f$ c) $f - g$ d) $g - f$ e) $f + f$ f) $g - g$

2) If $f(x) = x^2 - 3$ and $g(x) = \dfrac{-6}{x-2}$,

a) Determine $(f+g)(4)$

b) What is the domain of $(f+g)(x)$ and $(f-g)(x)$?

3) Make a reasonable sketch of the graph of $f+g$ and $f-g$, where $0 \le x \le 6$, for the function shown.



4) If $f(x) = \dfrac{1}{3x+4}$ and $g(x) = \dfrac{1}{x-2}$,

a. What is $f + g$? b. What is the domain of $f+g$?

c. What is $(f+g)(8)$? d. What is $(f-g)(8)$?

5) Given $f(x)$ and $g(x)$, write the function $y = f(x) + g(x)$ as a function of x.

a) $f(x) = 2x+3; g(x) = x-5$ b) $f(x) = x^2 - 4; g(x) = 2x+3$

c) $f(x) = x^2 + x - 1; g(x) = x^2 - 4x + 3$ d) $f(x) = (x-2)^2; g(x) = (2x-1)^2$

6) Given $f(x)$ and $g(x)$, write the function $y = f(x) - g(x)$ as a function of x.

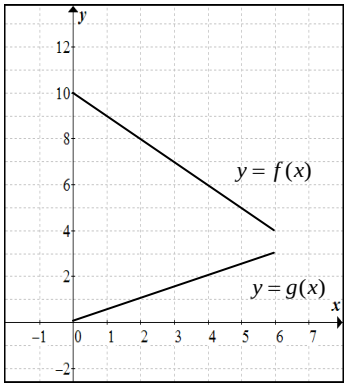
a) $f(x) = x+3; g(x) = 2-3x$ b) $f(x) = x^2 + 5x; g(x) = x^2 - x - 2$

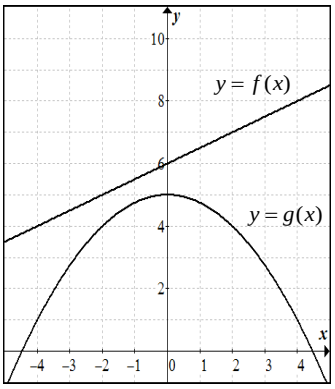
c) $f(x) = (x+1)^2; g(x) = (x-1)^2$ d) $f(x) = (2x-3)^2; g(x) = (3x+2)^2$

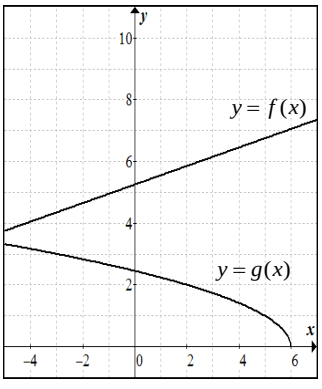
7) Copy each graph, and use the method of adding ordinates or subtract ordinates to draw the graph of

i) $y = f(x) + g(x)$ ii) $y = f(x) - g(x)$

a) b) c)







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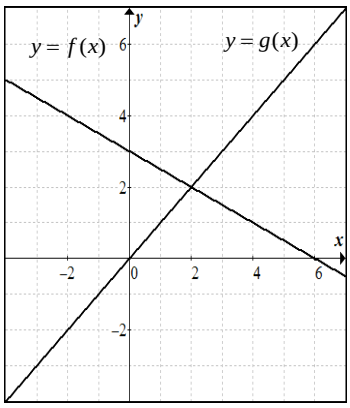
8) Given $f(x) = x$ and $g(x) = x + 1$
   a) Write the function $y = f(x) + g(x)$ as a function of *x*.
   b) Graph $y = f(x)$ and $y = g(x)$ on the same grid.
   c) Use the graphs in part b) to draw the graph of $y = f(x) + g(x)$.
   d) What is the domain of $y = f(x) + g(x)$?

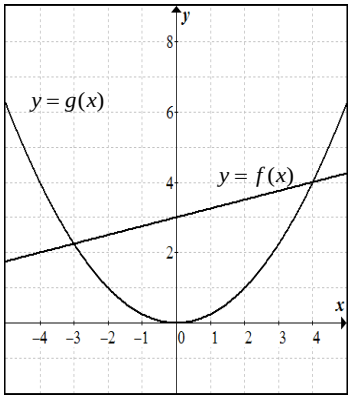
9) Given $f(x) = 2x + 4$ and $g(x) = x + 1$
   a) Write the function $y = f(x) - g(x)$ as a function of *x*.
   b) Graph $y = f(x)$ and $y = g(x)$ on the same grid.
   c) Use the graphs in part b) to draw the graph of $y = f(x) - g(x)$.
   d) What is the domain of $y = f(x) - g(x)$?

10) Copy each graph, and use the method of adding ordinates or subtract ordinates to draw the graph of
   i) $y = f(x) + g(x)$ ii) $y = f(x) - g(x)$

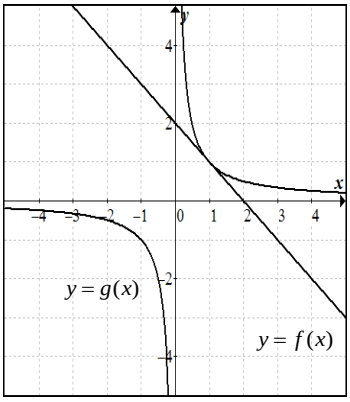
a)



b)



c)



11) Given $f(x) = x^2 - 4$ and $g(x) = 2x$, draw the graph of each function.
   a) $y = f(x)$ b) $y = g(x)$ c) $y = f(x) + g(x)$
   d) $y = f(x) - g(x)$ e) $y = f(x) \cdot g(x)$ f) $y = \dfrac{f(x)}{g(x)}$

12) Given $f(x) = \sqrt{x}$ and $g(x) = x$, draw the graph of each function.
   a) $y = f(x)$ b) $y = g(x)$ c) $y = f(x) + g(x)$
   d) $y = f(x) - g(x)$ e) $y = f(x) \cdot g(x)$ f) $y = \dfrac{f(x)}{g(x)}$

13) Draw the graph of $y = \dfrac{2x}{x^2 + 1}$. Use this graph to draw the graph of each function.
   a) $y = x + 3 + \dfrac{2x}{x^2 + 1}$ b) $y = \dfrac{2x}{x^2 + 1} - x$

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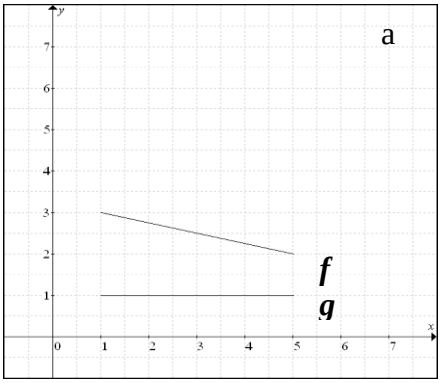
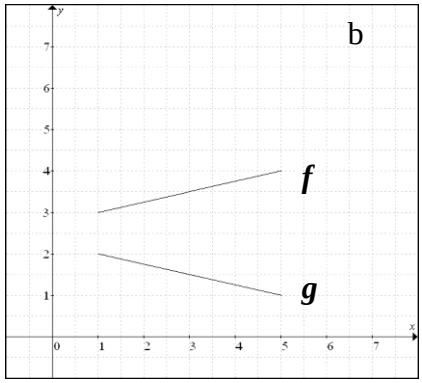
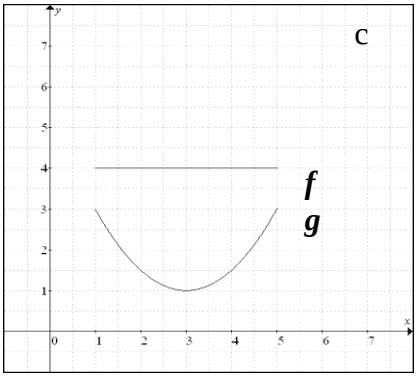
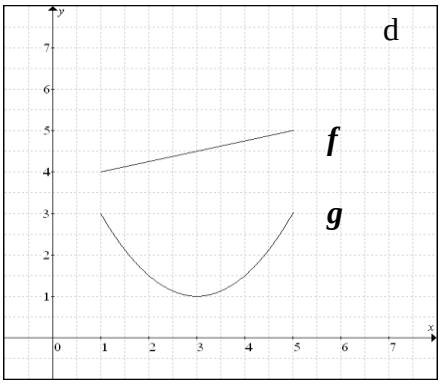
14) When a driver of a vehicle observes an obstacle in the vehicle's path, the driver reacts to apply the brakes and bring the vehicle to a complete stop. The distance that the vehicle travels while coming to a stop is a combination of the reaction distance, *r*, in metres, given by $r(x) = 0.21x$, and the braking distance, *b*, also in metres, given by $b(x) = 0.006x^2$. The speed of the vehicle is *x* km/h. Determine the stopping distance of the vehicle as a function of its speed, and calculate the stopping distance if the vehicle is travelling at 90 km/h.

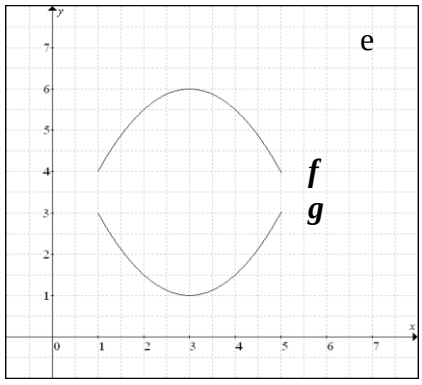
15) Given the graphs of f and g, sketch the graphs of i) f + g ii) f - g

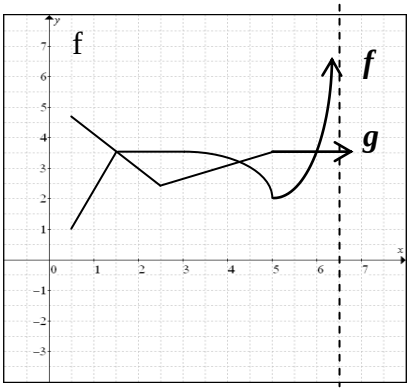












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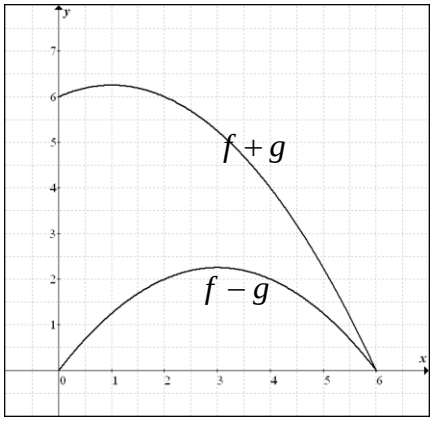
Answers

1a) {(-4,6),(-2,5),(1,5),(4,10)} b) {(-4,6),(-2,5),(1,5),(4,10)} c) {(-4,2),(-2,3),(1,1),(4,2)}

d) {(-4,-2),(-2,-3),(1,-1),(4,-2)} e) {(-4,8),(-2,8),(1,6),(3,10),(4,12)} f) {(-4,0),(-2,0),(0,0),(1,0),(2,0),(4,0)}

2a) 10 b) $x \neq 2, x \in R$

3)



4a) $\frac{2(2x+1)}{3x^2-2x-8}$ b) $x \neq \frac{-4}{3}, 2, x \in R$ c) $\frac{17}{84}$ d) $\frac{-11}{84}$

5a) $3x-2$ b) x^2+2x-1 c) $2x^2-3x+2$

d) $5x^2-8x+5$

6a) $4x+1$ b) $6x+2$ c) $4x$ d) $-5x^2-24x+5$

8a) $y=2x+1$ d) R 9a) $y=x+3$ d) R

14) The stopping distance can be defined by the function $s(x)=0.006x^2+0.21x$. If the vehicle is travelling at 90 km/h, the stopping distance is 67.5 m.

15)