Solving Logarithmic Equations

Date:

(a) Making both sides of the equation with a single log of the same base.

(1) When each term of an equation is a logarithm of the same base, apply laws of logarithm to make each side as a single logarithm.
(2) From (1), we have the form:
$\log_a A = \log_a B \Rightarrow A = B$
(3) Solve for the unknown in $A=B$.
(4) Check each root in the original equation. Reject any root that makes logarithm of a negative number.

Example 1

If $\log_a N = 3\log_a x + \log_a y - \log_a \sqrt{z}$, solve for N in terms of x, y, and z.

Example 2

Solve for x: $\log x - \frac{1}{3}\log 27 = \log 5$

Example 3

Solve for x: $\log_4(x+2) + \log_4(x-3) = \log_4 9$

(b) Solving logarithmic equations with constant terms

(1) Collect all the terms of logarithms with same base on one side, and make them as a single logarithm.
(2) Write the equation in the form $\log_a N$=number.
(3) Change to exponential form, and solve the equation.
(4) Check each root in the original equation. Reject any root that makes logarithm of a negative number.

Example 4
Solve for x: $\log_4 x + \log_4 (x+6) = 2$

Example 5
Solve for x: $\log_3 80x - 2 = \log_3 (x^2 - 1)$

Example 6
Determine the points of intersection of the curves $y = \log_{10}(x-2)$ and $y = 1 - \log_{10}(x+1)$.

Example 7
Explain why there are no solutions to the equations $\log_5(-125) = x$ and $\log_{-2} 16 = x$

Homework:
p.391 #1 – 3, 5, 6, 9, 11 (omit 11b)
7.3 #1-4, 6-8

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1) Solve each of the following exponential equation.

a) $3(16)^x + 2(81)^x = 5(36^x)$ b) $2^{\frac{2x}{3}+1} - 3(2)^{\frac{x}{3}} - 20 = 0$ c) $3^{2x+2} - 3^{x+3} - 3^x + 3 = 0$

d) $\frac{3^{\sqrt{12x}} + 3}{4} = 3^{\sqrt{3x}}$ e) $(x^2 - x - 1)^{x+2} = 1$

2) Solve each of the following equation for *x*.

a) $\log_5 x + \log_{\sqrt{5}} x + \log_{25} x = 10.5$ b) $2\log_2 x + 8\log_x 2 = 17$

c) $\log_{\sqrt{3}} x + \log_3 x + \log_{3\sqrt{3}} x = 11$ d) $\log_x 81 + \log_{\sqrt{x}} 3\sqrt{3} = 7$

e) $\log_5(x-2) + \log_{\sqrt{5}}(x-2) + \log_{0.2}(x-2) = 4$

f) $(\log_2 x)(\log_3 x)(\log_5 x) = 2[(\log_2 x)(\log_3 x) + (\log_3 x)(\log_5 x) + (\log_5 x)(\log_2 x)]$

3) If $\log_2 3 = a$ and $\log_5 6 = b$, find $\log_{10} 9$ in terms of *a* and *b*.

4) Solve the system of equations

$$\log_9 x + \log_9 y + \log_3 z = 2$$
$$\log_{16} x + \log_4 y + \log_{16} z = 1$$
$$\log_5 x + \log_{25} y + \log_{25} z = 0$$

5) Determine $\log_{10}(\tan 1^\circ) + \log_{10}(\tan 2^\circ) + \log_{10}(\tan 3^\circ) + \ldots + \log_{10}(\tan 89^\circ)$

6) If $\log_x w = 24, \log_y w = 40$ and $\log_{xyz} w = 12$, find $\log_z w$

7) Solve each of the following equation for *x*.

a) $\log_6 x - \log_6(x-1) = 1$ b) $\log_3(x-2) + \log_3 x = 1$ c) $\log_2(3x-1) + \log_2(2x-1) = 0$

d) $\log(24 + x^2) = 2\log(3 - x)$ e) $\log_2(x+24) - \log_2 x = 3\frac{1}{2}$ f) $\log(x+1) + \log(x-2) = 1$

g) $\log_5(2x+1) + \log_5(x-1) = 1$ h) $\frac{\log(x^3 - 4x)}{\log(x-2)} = 3$ i) $\log(20x^2 - 1) - \log(2x-1) = 1 + \log(x-5)$

j) $(\log x)^2 - \log x^3 = 10$ k) $\log_2[\log_2(\log_2 x)] = 0$ l) $x^{\log_{10} x} = 10$ m) $\log_2[\log_x 3] = \frac{-1}{2}$

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8) Solve each of the following to 2 decimal places.

a) $10^{5^x} = 3$ b) $4^x - 2^{x+1} - 3 = 0$ c) $\log_2 x + 8\log_x 2 = 9$

9) If $\log_a x = m$, find the value of each of following in terms of *m*:

a) $\log_{a^2} x$ b) $\log_{\frac{1}{a}} x$ c) $\log_x a$ d) $\log_{x^3} a$ e) $\log_{a^2} x^3$ f) $\log_{\frac{1}{x^2}} \frac{1}{a}$

10) Simplify each of the following.

a) $3^{\log_3 27} + 10^{\log_{10} 15}$ b) $5^{\log_5 8} - 3^{(\log_3 5 + \log_3 7)}$ c) $2^{4\log_4 5} + 3^{2\log_3 10}$ d) $5^{\log_{25} 64} + 3^{\log_{\frac{1}{3}} 5}$

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Answers:

1a) 0 & $\frac{1}{2}$ b) 6 c) 1 & -2 d) $\frac{1}{3}$ & 0 e) 2 & -1 & -2

2a) 125 b) $\sqrt{2}$ & 256 c) 27 d) 3 e) 27 f) 900

3) $\frac{2ab}{a+b+1}$ 4) $x = \frac{1}{6}, y = \frac{8}{3}, z = \frac{27}{2}$ 5) 0

6) 60 7a) $\frac{6}{5}$ b) 3 c) $\frac{5}{6}$ d) $\frac{-15}{6}$ e) $\frac{24}{2^{3.5} - 1}$ f) 4 g) 2 h) No Solution i) No Solution

j) $\frac{1}{100}$, 100000

k) 4 l) 10 & $\frac{1}{10}$ m) $3^{\sqrt{2}}$ 8a) -0.46 b) 1.58 c) 2 & 256

9a) $\frac{m}{2}$ b) -*m* c) $\frac{1}{m}$ d) $\frac{1}{3m}$ e) $\frac{3m}{2}$ f) $\frac{1}{2m}$

10a) 42 b) -27 c) 125 d) $8\frac{1}{5}$