Change of Base

Date:

(1) $\log_a b = \dfrac{\log_x b}{\log_x a} = \dfrac{\log b}{\log a}$

(2) $\log_a b = \dfrac{1}{\log_b a}$

(3) In particular, $\log b = \dfrac{\log b}{\log 10}$

Example 1

Evaluate to 2 decimal places:

a) $\log_2 23$

b) $\log_4 4.7$

Example 2

Prove that $\log_t b = \dfrac{1}{\log_b t}$.

Example 3

Show that $\dfrac{1}{\log_3 a} + \dfrac{1}{\log_4 a} = \dfrac{1}{\log_{12} a}$.

Example 4

If $a^2 + b^2 = 14ab$, where $a > 0, b > 0$, show that $\log\left(\frac{a+b}{4}\right) = \frac{1}{2}(\log a + \log b)$

Example 5 P.286 #8

If $\log_a b = p^3$ and $\log_b a = \frac{4}{p^2}$, show that $p = \frac{1}{4}$.

Example 6

If $y = (\log_2 3)(\log_3 4)(\log_4 5).....(\log_{31} 32)$, **calculate the value of *y*.**

Homework:
WS: Change of Base

Change of Base

Date:

(1) $\log_a b = \dfrac{\log_x b}{\log_x a} = \dfrac{\log b}{\log a}$	(2) $\log_a b = \dfrac{1}{\log_b a}$	(3) In particular, $\log b = \dfrac{\log b}{\log 10}$

1. Use your calculator to find the value of each of the following, correct to three decimal places.
   a) $\log_5 21$ b) $\log_7 124$ c) $\log_6 3.24$ d) $\log_4 4.7$

2. Show that each of the following statement is true.
   a) $\dfrac{1}{\log_5 a} + \dfrac{1}{\log_3 a} = \dfrac{1}{\log_{15} a}$ b) $\dfrac{1}{\log_8 a} - \dfrac{1}{\log_2 a} = \dfrac{1}{\log_4 a}$
   c) $\dfrac{2}{\log_6 a} = \dfrac{1}{\log_{36} a}$ d) $\dfrac{2}{\log_8 a} - \dfrac{4}{\log_2 a} = \dfrac{1}{\log_4 a}$

3. Describe the changes to the graph of $y = \log_3 x$ when x is replaced by x^2.

4. For $a > 1, b > 1,$ show that $(\log_a b)(\log_b a) = 1$

5. If $a^2 + b^2 = 23ab,$ where $a > 0, b > 0$, show that $\log\left(\dfrac{a+b}{5}\right) = \dfrac{1}{2}(\log a + \log b)$.

6. For $a > 0, a \neq 1, x > 0,$ prove that $\log_a \dfrac{1}{x} = \log_{\frac{1}{a}} x$.

7. If $\log_a b = p^3$ and $\log_b a = \dfrac{4}{p^2}$, show that $p = \dfrac{1}{4}$.

8. If $a^3 - b^3 = 3a^2b + 5ab^2$, where $a > 0, b > 0,$ show that $\log\left(\dfrac{a-b}{2}\right) = \dfrac{1}{3}(\log a + 2\log b)$.

Answers

1a) 1.892 b) 2.477 c) 0.656 d) 1.116

3) Graph is reflected in y – axis.