

Logarithmic Function vs Exponential Function

To change the exponential function to logarithmic function, or vice versa, use the fact that: $y = \log_b x \Leftrightarrow x = b^y$.

$$2^4 = 16 \Leftrightarrow \log_2 16 = 4$$

e.g. $3^2 = 9 \Leftrightarrow \log_3 9 = 2$

$$4^{-2} = \frac{1}{16} \Leftrightarrow \log_4 \frac{1}{16} = -2$$

Common Logarithm

The **common logarithm** is the logarithm with base as 10. This base 10 is usually omitted.

$$\log_{10} 1000 = 3 \Leftrightarrow \log 1000 = 3;$$

e.g. $\log_{10} \frac{1}{100} = -2 \Leftrightarrow \log \frac{1}{100} = -2$

Example 2

Change to logarithmic form

(a) $27^{\frac{2}{3}} = 9$

(b) $2^{-3} = \frac{1}{8}$

Example 3

Change to exponential form

(a) $\log_{\frac{1}{3}} 9 = -2$

(b) $\log_9 27 = \frac{3}{2}$

Example 4

Use your calculator to find the value of the following:

(a) $\log_{10} 500$

(b) $\log 0.0231$

Basic Properties of Logarithm

- 1) $\log_a 1 = 0$ 2) $\log_a a = 1$ 3) $\log_a a^x = x$ 4) $a^{\log_a x} = x$ 5) $\log_a b = \frac{\log b}{\log a}$
(Change of Base)

Example 5

Evaluate

- (a) $\log_3 81$ (b) $\log_2 \frac{1}{8}$ (c) $\log_2 \sqrt[4]{32}$

Example 6

Solve for x

- (a) $\log_{\frac{1}{4}} x = -2$ (b) $\log_x 27 = 3$ (c) $\log_4 \left(\frac{1}{4}\right) = x$

Example 7

If $\log_3(\log_4 a^3) = 1$, find a .

Example 8

Solve the equations for the point of intersection of the graphs of $y = \log_2(2x)$ and $y = \log_4 x$.