The Remainder Theorem tells us that if the remainder is zero on division by \((x - p)\), then \(f(p) = 0\). If the remainder is zero, then \((x - p)\) divides evenly into \(f(x)\), and \((x - p)\) is a factor of \(f(x)\). Conversely, if \(x - p\) is a factor of \(f(x)\), then the remainder \(f(p)\) must equal zero. These two statements give us the Factor Theorem, which is an extension of the Remainder Theorem.

**Definition**

\((x - a)\) is a factor of \(f(x)\) if and only if \(f(a) = 0\).

**Example 1**

Factor \(2x^2 - 5x - 3\)

**Example 2**

a) Show that \(x - 2\) is a factor of \(x^3 - 3x^2 + 5x - 6\).
b) Factor \(x^3 - 3x^2 + 5x - 6\)

**Example 3**

Factor \(x^3 - x^2 - 14x + 24\)

**Example 4**

Factor

(a) \(x^3 - y^3\)  
(b) \(27y^3 + 64\)  
(c) \(5u^3 - 40(x + y)^3\)

**The Sum and Difference of Cubes**

\[x^3 - y^3 = (x - y)(x^2 + xy + y^2)\]

\[x^3 + y^3 = (x + y)(x^2 - xy + y^2)\]
Example 5
Prove that \((x - a)\) is a factor of \(x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc\)

Example 6
Given the function \(y = x^3 - 2x^2 - x + 2\)

i) State the intercepts (if any)
ii) State the degree
iii) Sketch and describe the roots.
iv) Determine the symmetry

Homework:
p.102 #1 – 4, 6, 9, 10, 12 – 15, 20