Mathematics 9–12

Binomials Behaving Badly

#	What doesn't work.	Why it doesn't.
1.	$(a+b)^2 \neq a^2+b^2$	$(2+3)^2 = 5^2 = 25$ but… $2^2+3^2 = 4+9 = 13$ However, it is true that: $(a+b)^2 = (a+b)(a+b) = a^2+2ab+b^2$
2.	$\sqrt{a+b} \neq \sqrt{a}+\sqrt{b}$	$\sqrt{9+16} = \sqrt{25} = 5$ but… $\sqrt{9}+\sqrt{16} = 3+4 = 7$
3.	$\sqrt{a^2+b^2} \neq a+b$	$\sqrt{5^2+12^2} = \sqrt{25+144} = \sqrt{169} = 13 \neq 5+12$ (Similar to #2 above.)
4.	$\frac{1}{a+b} \neq \frac{1}{a}+\frac{1}{b}$	$\frac{1}{2+4} = \frac{1}{6}$ but… $\frac{1}{2}+\frac{1}{4} = \frac{2}{4}+\frac{1}{4} = \frac{3}{4}$ However, it is true that: $\frac{1}{a}+\frac{1}{b} = \frac{b}{ab}+\frac{a}{ab} = \frac{a+b}{ab}$
5.	$(a+b)^{-1} \neq a^{-1}+b^{-1}$	See #1 and #4 above.
6.	$\lvert a+b\rvert \neq \lvert a\rvert+\lvert b\rvert$	$\lvert -9+7\rvert = \lvert -2\rvert = 2$ but… $\lvert -9\rvert+\lvert 7\rvert = 9+7 = 16$
7.	$\sin(a+b) \neq \sin a+\sin b$	$\sin(30°+60°) = \sin 90° = 1$ but… $\sin 30°+\sin 60° = \frac{1}{2}+\frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$
8.	$\cos(a+b) \neq \cos a+\cos b$ and $\tan(a+b) \neq \tan a+\tan b$	Similar to #7 above.
9.	$\frac{a+b}{c+d} \neq \frac{a}{c}+\frac{b}{d}$	$\frac{2+3}{4+6} = \frac{5}{10} = \frac{1}{2}$ but… $\frac{2+3}{4+6} = \frac{2}{4}+\frac{3}{6} = \frac{1}{2}+\frac{1}{2} = 1$
10.	$\frac{a+b}{a+c} \neq \frac{1+b}{1+c}$	$\frac{2+3}{2+8} = \frac{5}{10} = \frac{1}{2}$ but… $\frac{1+3}{1+8} = \frac{4}{9}$
11.	$\frac{a+b}{a+c} \neq 1+\frac{b}{c}$	$\frac{2+3}{2+8} = \frac{5}{10} = \frac{1}{2}$ but… $1+\frac{3}{8} = \frac{8}{8}+\frac{3}{8} = \frac{11}{8}$
12.	$\frac{1}{a^{-1}+b^{-1}} \neq a+b$	$\frac{1}{2^{-1}+4^{-1}} = \frac{1}{\frac{1}{2}+\frac{1}{4}} = \frac{1}{\frac{2}{4}+\frac{1}{4}} = \frac{1}{\frac{3}{4}} = 1 \div \frac{3}{4} = 1 \times \frac{4}{3} = \frac{4}{3} \neq 2+4$ This is actually an example of #4 above and thus: $\frac{1}{a^{-1}+b^{-1}} = \frac{1}{\frac{1}{a}+\frac{1}{b}} = \frac{1}{\frac{a+b}{ab}} = \frac{ab}{a+b}$
13.	In the sum: $\frac{1}{x+2}+\frac{1}{x+4}$ the common denominator is neither $x+4$ nor $x+6$ nor is it $x+8$	The common denominator of two dissimilar binomials is not usually another binomial. Instead: $\frac{1}{x+2}+\frac{1}{x+4} = \frac{x+4}{(x+2)(x+4)}+\frac{x+2}{(x+2)(x+4)} = \frac{2x+6}{(x+2)(x+4)}$
14.	If: $\frac{1}{a}+\frac{1}{a} = \frac{1}{b}$ then the next step is **<u>not</u>** to take the reciprocal of both sides to give: $a+a = b$	$\frac{1}{4}+\frac{1}{4} = \frac{1}{2}$ does not mean that: $4+4 = 2$