Series and Sequences

Math Club Junior

Arithmetic Sequences

- A sequence of numbers such that the difference between the consecutive terms is constant
- $a_{n+1} = a_n + d$ $\{a, a + d, a + 2d, a + 3d, \ldots\}$
- Common difference, d
- E.g. 1, 4, 7, 10, 13, 16 $d = 3$

To find any term: $a_n = a_1 + (n-1)d$ where a_1 is the first term of the sequence, n is the number of term to find

Geometric Sequence

- A sequence of numbers where each term is found by multiplying the previous one by a fixed ratio
- Common ratio, r   r is non-zero
- E.g. 2, 4, 8, 16, 32, 128, 256, … r = 2

To find any term: $a_n = a_1 r^{n-1}$ where a_1 is the first term of the sequence, n is the number of the term to find

Arithmetic and Geometric Series

- Finding the sum of a finite number of terms in a sequence
- Arithmetic
  - $S_n = n(a_1 + a_n) / 2$ $S_n = n(2a_1 + (n - 1)d) / 2$
- Geometric
  - $S_n = (a_{n+1} - a_1) / (r - 1)$ $S_n = a(r^n - 1) / (r - 1)$
  - If it's an infinite number of terms and $r < 1$
    - $S = a / (1 - r)$

S_n is the sum of n terms, a_1 is the first term, a_n is the n^{th} term

Sigma Notation

$$\sum_{i=1}^{n} a_i$$

- Put the sigma symbol in front of the general term
- *n* is the last term (upper limit of summation)
- *i* = is the first term (lower limit of summation)
  - E.g. *i = 3,* the sequence starts on a_3
- E.g.

$$1+2+3+4+5=\sum_{i=1}^{5} i \qquad 4+8+16+32+64=\sum_{i=2}^{6} 2^i$$

1. $\sum_{j=1}^{4} 3j = 3(1)+3(2)+3(3)+3(4)$
2. $\sum_{k=3}^{6} k = 3+4+5+6$
3. $\sum_{i=-1}^{2} (i+3) = (-1+3)+(0+3)+(1+3)+(2+3)$
4. $\sum_{i=0}^{2} 2^i = 2^0+2^1+2^2$
5. $\sum_{j=5}^{6} \left(x^j-2j+1\right) = \left(x^5-2(5)+1\right)+\left(x^6-2(6)+1\right)$
6. $\sum_{k=1}^{7} 4 = 4+4+4+4+4+4+4$

Cool Formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = (\frac{n(n+1)}{2})^2$$

Questions

1. An arithmetic series has 38 terms, the first is -28 and the last is 305. Find the sum of the series   5263
2. A geometric series with 9 terms has a common ratio of 3 and a sum of 757. Find the first 3 terms.   1/13, 3/13, 9/13
3. -3 + 2 + 7 + … = 1943. How many terms are there?   29
4. An arithmetic series whose first term is -37 and last term is 131 has a sum of 2679. Find the common difference. 3
5. An arithmetic series' first term is -92, fourth term is -101, and has a sum of -6665. How many terms are there?   43
6. Evaluate $\sum_{i=1}^{\infty} 2(3)^{-i}$ 1

Questions

1. (Cayley 1989, #24) $1^4 + 2^4 + 3^4 + 4^4 + \cdots + n^4$ is given by the expression $\dfrac{6n^5 + an^4 + bn^3 - n}{30}$ What's a-b?

2. (Cayley 2016, #9) Grace writes a sequence of 20 numbers. The first number is 43 and each number after the first is 4 less than the number before it, so her sequence starts 43, 39, 35, ... How many of the numbers that Grace writes are positive?

3. (Cayley 2010, #22) A sequence consists of 2010 terms. Each term after the first is 1 larger than the previous term. The sum of the 2010 terms is 5307. When every second term is added up, starting with the first term and ending with the second last term, the sum is?

Substituting Terms to Create a Arth/Geo Sequence

If there is a general pattern for each term, we can gather terms together and notate them as one term for another sequence

EG: If we subtract 5 from each general term from a sequence $\{ a_1, a_2, \ldots a_n \}$,

we will get $\{ a_1 - 5, a_2 - 5, \ldots a_n - 5\}$. We can let a new sequence $b_n = a_n - 5$.

Then $\{ a_1 - 5, a_2 - 5, \ldots a_n - 5\} = \{ b_1, b_2, \ldots b_n \}$

EG: If we multiply each general term from a sequence $\{ a_1, a_2, \ldots a_n \}$ by 2,

we will get $\{ 2a_1, 2a_2, \ldots 2a_n \}$. We can let a new sequence $b_n = 2a_n$.

Then $\{ 2a_1 , 2a_2 , \ldots 2a_n\} = \{ b_1, b_2, \ldots b_n \}$

Problem 1

1. Given sequence $\{a_n\}$ with $a_1 = 2$ that satisfies $a_{n+1} = 2a_n + 3\times 2^n$， $a_1 = 2$， determine the general term a_n.

Problem 2

7. Given sequence $\{a_n\}$ with $a_1 = 3$ that satisfies $a_{n+1} = 3a_n + 2\times 3^n + 1$. Determine the general term a_n.

Problem 3

5. Given sequence $\{a_n\}$ with $a_1 = 6$ that is satisfies $a_{n+1} = 2a_n + 3 \times 5^n$. Determine the general a_n.

More Series and Sequences Questions

1. (Cayley 2005, #16) The non-negative difference between two numbers a and b is a − b or b − a, whichever is greater than or equal to 0. For example, the non-negative difference between 24 and 64 is 40. In the sequence 88, 24, 64, 40, 24, . . ., each number after the second is obtained by finding the non-negative difference between the previous 2 numbers. The sum of the first 100 numbers in this sequence is?
2. (Cayley 2001, #14) The sequences 3, 20, 37, 54, 71, ... and 16, 27, 38, 49, 60, 71, ... each have 71 as a common term. The next term that these sequences have in common is?