Math Club

Series and Sequences - Solutions

NOTE: If you don't know how to solve quadratics (taught in Gr 10 Math) do not do #3 and 5.

1. **An arithmetic series has 38 terms, the first is -28 and the last is 305. Find the sum of the series**

$n = 38 \quad a_1 = -28 \quad a_{38} = 306$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{38} = \frac{n}{2}(a_1 + a_{38})$$

$$S_{38} = \frac{38}{2}(-28 + 305)$$

$S_{38} = 5263$

2. **A geometric series with 9 terms has a common ratio of 3 and a sum of 757. Find the first 3 terms.**

$n = 9 \quad r = 3 \quad S_9 = 757$

$$S_n = \frac{a_1(r^n - 1)}{r - 1}$$

$$S_9 = \frac{a_1(3^9 - 1)}{3 - 1}$$

$$757 = \frac{a_1(3^9 - 1)}{3 - 1}$$

$$a_1 = \frac{1}{13} \qquad Since\ r = 3, a_2 = \frac{3}{13} \qquad a_3 = \frac{9}{13}$$

3. **-3 + 2 + 7 + … = 1943. How many terms are there?**

$d = 5 \quad S_n = 1943 \quad a_1 = -3$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$$1943 = \frac{n}{2}(2 \times -3 + (n-1) \times 5)$$

$$1943 \times 2 = \frac{n}{2} \times 2(-6 + 5n - 5)$$

$$3886 = n(5n - 11)$$

$$3886 = 5n^2 - 11n$$

$$5n^2 - 11n - 3886 = 0$$

use either the quadratic formula or your calcultor for the answer

$n = 29$

4. **An arithmetic series whose first term is -37 and last term is 131 has a sum of 2679. Find the common difference.**

$a_1 = -37 \quad a_n = 131 \quad S_n = 2679$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$2679 = \frac{n}{2}(-37 + 131)$$

$$n = 57$$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$$2679 = \frac{57}{2}(2 \times (-37) + (57 - 1)d)$$

$$2679 = \frac{57}{2}(-74 + (56)d)$$

$$d = 3$$

5. An arithmetic series' first term is -92, fourth term is -101, and has a sum of -6665. How many terms are there?

$a_1 = -92 \quad a_4 = -101 \quad S_n = -6665$
$a_1 = -92 \quad a_4 = -101 \quad S_n = -6665$
$a_n = a_1 + (n-1)d$
$a_4 = a_1 + (4-1)d$
$-101 = -92 + (4-1)d$
$d = -3$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$
$$-6665 = \frac{n}{2}\big(2 \times (-92) + (n-1) \times (-3)\big)$$
$$-6665 \times 2 = \frac{n}{2} \times 2(-184 - 3n + 3)$$
$$-13330 = n(-3n - 181)$$
$$-13330 = -3n^2 - 181n$$
$$3n^2 + 181n - 13330 = 0$$
use either the quadratic formula or your calcultor for the answer
$n = 43$

6. Evaluate

$$\sum_{i=1}^{\infty} \mathbf{2(3)^{-i}}$$

$$\sum_{i=1}^{\infty} 2(3)^{-i} = \sum_{i=1}^{\infty} 2\left(\frac{1}{3}\right)^{i}$$

Since the series is infinite and $r = \frac{1}{3}$ *(which is greater than* -1 *and less than* 1*), you can use:*

$$S = \frac{a}{1-r}$$
$$S = \frac{2\left(\frac{1}{3}\right)^{1}}{1 - \frac{1}{3}}$$
$$S = \frac{\frac{2}{3}}{\frac{2}{3}}$$
$$S = 1$$