Problem 1: $a_{n+1} = 2a_n + 3 \times 2^n$, $a_1 = 2$

$$\frac{a_{n+1}}{2^{n+1}} = \frac{2a_n}{2^{n+1}} + 3\frac{2^n}{2^{n+1}}$$

$$\frac{a_{n+1}}{2^{n+1}} = \frac{a_n}{2^n} + \frac{3}{2}$$

Let $b_n = \frac{a_n}{2^n}$

$$b_{n+1} = b_n + \frac{3}{2}$$

$$b_n = b_{n-1} + \frac{3}{2}$$

Since b_n is an arithmetic sequence, we can rewrite b_n as $b_1 + (n-1)d$

$$b_n = b_1 + ((n-1)-1)\frac{3}{2} + \frac{3}{2}$$

$$b_n = b_1 + (n-1)\frac{3}{2}$$

$$b_1 = \frac{a_1}{2^1}$$

$$= \frac{2}{2}$$

$$= 1$$

$$b_n = 1 + \frac{3}{2}(n-1)$$

$$\frac{a_n}{2^n} = 1 + \frac{3}{2}(n-1)$$

$$a_n = 2^n\left(1 + \frac{3}{2}(n-1)\right)$$

Problem 2: $a_{n+1} = 3a_n + 2 \times 3^n + 1 \, , \ a_1 = 3$

$$\frac{a_{n+1}}{3^{n+1}} = \frac{3a_n}{3^{n+1}} + 2 \times \frac{3^n}{3^{n+1}} + \frac{1}{3^{n+1}}$$

$$\frac{a_{n+1}}{3^{n+1}} = \frac{a_n}{3^n} + \frac{2}{3} + \frac{1}{3^{n+1}}$$

Let $b_n = \frac{a_n}{3^n}$

$$b_{n+1} = b_n + \frac{2}{3} + \frac{1}{3^{n+1}}$$

$$b_n = b_{n-1} + \frac{2}{3} + \frac{1}{3^n}$$

b_n is the sum of a arithmetic & geometric series.

$$b_n = b_1 + \frac{2}{3}(n-1) + \sum_{k=2}^{n} \frac{1}{3^k}$$

$$b_n = \frac{3}{3} + \frac{2}{3}(n-1) + \frac{\frac{1}{9}\left(1 - \frac{1}{3^n}\right)}{1 - \frac{1}{3}}$$

$$= 1 + \frac{2}{3}(n-1) + \frac{1}{6}\left(1 - \frac{1}{3^n}\right)$$

Q2 cont!

$$\frac{a_n}{3^n} = 1 + \frac{2}{3}n - \frac{2}{3} + \frac{1}{6} - \frac{1}{6} \times \frac{1}{3^n}$$

$$= \frac{1}{2} + \frac{2}{3}n - \frac{1}{6} \times \frac{1}{3^n}$$

$$a_n = 3^n\left(\frac{1}{2} + \frac{2}{3}n - \frac{1}{6} \cdot \frac{1}{3^n}\right)$$

$$= 3^n\left(\frac{1}{2} + \frac{2}{3}n\right) - \frac{1}{6}$$

Question 3: $a_{n+1} = 2a_n + 3 \times 5^n$, $a_1 = 6$

$$a_{n+1} - 5 \times 5^n = 2a_n + 3 \times 5^n - 5 \times 5^n$$

$$a_{n+1} - 5^{n+1} = 2a_n - 2 \times 5^n$$

Let $b_n = a_n - 5^n$

$b_{n+1} = 2b_n$

$b_n = 2b_{n-1}$

$b_n = b_1 \times 2^{n-1}$

$a_n - 5^n = (a_1 - 5^1) \times 2^{n-1}$

$a_n = 2^{n-1} + 5^n$