



Probability in Math Contests

Math Club Junior

Definition of Probability

- Theoretical Probability - $\frac{\text{\# of ways the event can occur}}{\text{total \# of possible outcomes}}$
- Experimental Probability - $\frac{\text{\# of trials in which the event occurred}}{\text{total \# of trials in the experiment}}$
- Subjective Probability - an estimate based on intuition
 - Little or no math needed
- Probability is a number between 0 and 1,
 - Probability of 0 would mean that the event never occurs
 - Probability of 1 would mean that the event always occurs

Symbols and Notations

- Sample Space (S) - all distinct possible outcomes
 - Probability of $A \rightarrow P(A) = n(A) / n(S) \leftarrow \# \text{ of possible outcomes}$
 $\uparrow \# \text{ of ways it can occur}$

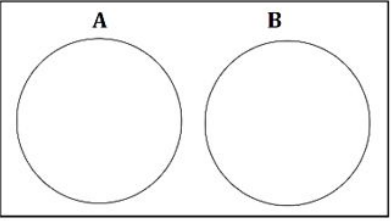
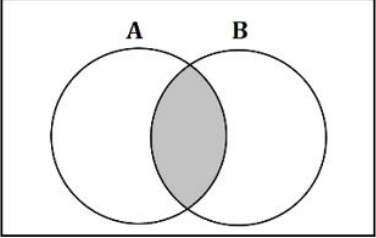
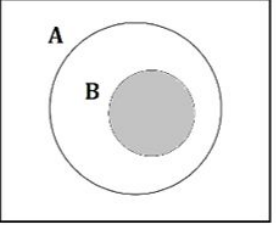
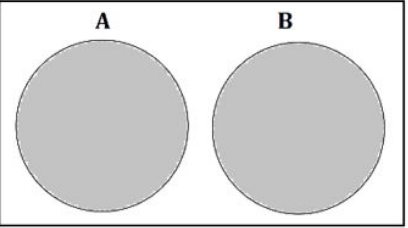
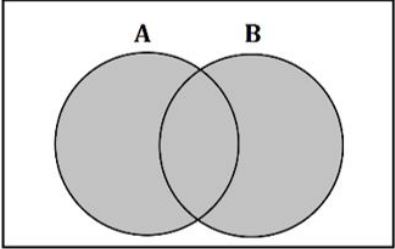
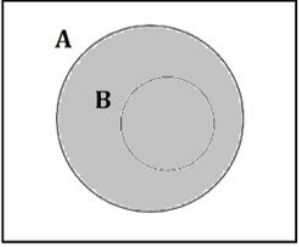
- E.g. sample space for rolling a die

$$S = \{1, 2, 3, 4, 5, 6\}$$

- What are the chances of rolling a 1 on a die?

$$P(\text{rolling a 1}) = n(\text{rolling a 1}) / n(S)$$

$$= 1 / 6$$

	Mutually Exclusive	Non-Mutually Exclusive	Subset
AND	 <p>$n(A \cap B) = 0$</p>	 <p>$n(A \cap B) = n(A \text{ and } B)$</p>	 <p>$n(A \cap B) = n(B)$</p>
OR	 <p>$n(A \cup B) = n(A) + n(B)$</p>	 <p>$n(A \cup B) = n(A) + n(B) - n(A \cap B)$</p>	 <p>$n(A \cup B) = n(A)$ $B \subseteq A$</p>

	Dependent	Independent
AND	$P(A \cap B) = P(A B) \times P(B)$ $= P(B A) \times P(A)$	$P(A \cap B) = P(A) \times P(B)$
GIVEN	$P(A B) = P(A \cap B) \div P(B)$	$P(A B) = P(A)$ $P(A) \text{ doesn't depend on } B$

Symbols and Notations

- Mutually Exclusive - events that cannot happen at the same time
- Non-Mutually Exclusive - events that can happen at the same time
- Independent Events - when one event has no effect on the probability of another
 - $P(A \text{ and } B) = P(A) \times P(B)$
- Dependent Events - when one event effects on the probability of another
- Conditional Probability - probability of a second event occurring given that the first event has already occurred
 - $P(A|B) = P(A \text{ and } B) \div P(B)$

Questions

1. The probability of failing your math quiz is 0.002 and the probability of failing your science quiz is 0.005. What's the probability that you'll fail both? 0.00001
2. A die is rolled. Find the probability that the number is odd **GIVEN** that the number is less than 4. 66.7%
3. A pair of dice is rolled. Find the probability that the numbers on the dice are equal given that the sum of the 2 numbers is even. 33.3%
4. You draw a card from 2 decks of cards. What is the probability that you:
 - a. Draw a club from deck 1 and a diamond from deck 2 1/16
 - b. Draw an ace from both deck 1 and deck 2 1/169
 - c. Draw a black card from deck 1 and a red ace from deck 2 1/52
5. The probability that a person votes for Trump is $\frac{1}{6}$ and the probability that a person completes a university degree is $\frac{1}{10}$. If these are independent, what's the probability that someone:
 - a. Votes for Trump and has a degree 1/60
 - b. Doesn't vote for Trump and doesn't have a degree 3/4
 - c. Votes for Trump or has a degree 1/4

Hard Questions

1. A test for ebola shows positive for people with the disease 85% of the time, but also positive for people who don't have ebola. 8% of the population has ebola. If you get a positive reaction to the test, what is the probability that you actually have ebola? (Hint: draw a tree diagram and include the probability on each branch) 78.7%
2. A small factory has 3 machines for decorating cake. The high speed machine decorates 60% of the cakes, but 5% of them are smudged. The medium speed machine decorates 30% of the cakes, but 3% of them are smudged. The low speed machine decorates 10% of the cakes, but 1% of them are smudged. If a cake comes out smudged, what's the probability that it came from:
 - a. The high speed machine 75%
 - b. The medium speed machine 22.5%
 - c. The low speed machine 2.5%