

1. The probability of failing your math quiz is 0.002 and the probability of failing your science quiz is 0.005. What's the probability that you'll fail both?

Since these events are independent,

$$P(A \cap B) = P(A) \times P(B)$$

$$\begin{aligned} P(\text{fail math quiz} \cap \text{fail science quiz}) &= P(\text{fail math quiz}) \times P(\text{fail science quiz}) \\ &= 0.002 \times 0.005 \\ &= 0.0001 \end{aligned}$$

2. A die is rolled. Find the probability that the number is odd GIVEN that the number is less than 4.

Since these are dependant,

$$P(A|B) = P(A \cap B) \div P(B)$$

$$P(\text{odd} | \text{less than 4}) = P(\text{odd} \cap \text{less than 4}) \div P(\text{less than 4})$$

$$\begin{aligned} \text{Count the numbers that are odd AND less than 4} &= \{1, 3\} \\ &= 2/6 \div 3/6 \\ &= 2/3 \end{aligned}$$

3. A pair of dice is rolled. Find the probability that the numbers on the dice are equal given that the sum of the 2 numbers is even.

Since these are dependant,

$$P(A|B) = P(A \cap B) \div P(B)$$

$$P(\text{equal} | \text{sum is even}) = P(\text{equal} \cap \text{sum is even}) \div P(\text{sum is even})$$

$$\text{Count all numbers that are equal and the sum is even} = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$$

6 total

$$\text{and the numbers with an even sum} = \{(1, 3), (1, 5), (3, 1), (5, 1), (3, 5), (5, 3), (1, 1), (3, 3), (5, 5), (2, 4), (2, 6), (4, 2), (6, 2), (4, 6), (6, 4), (2, 2), (4, 4), (6, 6)\}$$

18 total

$$\begin{aligned} P(\text{equal} | \text{sum is even}) &= P(\text{equal} \cap \text{sum is even}) \div P(\text{sum is even}) \\ &= 6/36 \div 18/36 \\ &= 1/3 \end{aligned}$$

4. You draw a card from 2 decks of cards. What is the probability that you:
a. Draw a club from deck 1 and a diamond from deck 2

Since these are independent,

$$P(A \cap B) = P(A) \times P(B)$$

$$\begin{aligned} P(\text{deck 1 club} \cap \text{deck 2 club}) &= P(\text{deck 1 club}) \times P(\text{deck 2 club}) \\ &= 13/52 \times 13/52 \\ &= 169/2704 \\ &= 1/16 \end{aligned}$$

b. Draw an ace from both deck 1 and deck 2

Since these are independent,

$$\begin{aligned}P(A \cap B) &= P(A) \times P(B) \\P(\text{deck 1 ace} \cap \text{deck 2 ace}) &= P(\text{deck 1 ace}) \times P(\text{deck 2 ace}) \\&= \frac{4}{52} \times \frac{4}{52} \\&= \frac{16}{2704} \\&= \frac{1}{169}\end{aligned}$$

c. Draw a black card from deck 1 and a red ace from deck 2

Since these are independent,

$$\begin{aligned}P(A \cap B) &= P(A) \times P(B) \\P(\text{deck 1 black} \cap \text{deck 2 red ace}) &= P(\text{deck 1 black}) \times P(\text{deck 2 red ace}) \\&= \frac{26}{52} \times \frac{2}{52} \\&= \frac{52}{2704} \\&= \frac{1}{52}\end{aligned}$$

5. The probability that a person votes for Trump is $\frac{1}{6}$ and the probability that a person completes a university degree is $\frac{1}{10}$. If these are independent, what's the probability that someone:

a. Votes for Trump and has a degree

Since these are independent,

$$\begin{aligned}P(A \cap B) &= P(A) \times P(B) \\P(\text{Trump} \cap \text{degree}) &= P(\text{Trump}) \times P(\text{degree}) \\&= \frac{1}{6} \times \frac{1}{10} \\&= \frac{1}{60}\end{aligned}$$

b. Doesn't vote for Trump and doesn't have a degree

Since these are independent,

$$\begin{aligned}P(A \cap B) &= P(A) \times P(B) \\P(\text{not Trump} \cap \text{no degree}) &= P(\text{not Trump}) \times P(\text{no degree}) \\&= \frac{5}{6} \times \frac{9}{10} \\&= \frac{45}{60} \\&= \frac{3}{4}\end{aligned}$$

c. Votes for Trump or has a degree $\frac{1}{4}$

Use this formula,

$$\begin{aligned}P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\P(\text{Trump or degree}) &= P(\text{Trump}) + P(\text{degree}) - P(\text{Trump and degree}) \\&= \frac{1}{6} + \frac{1}{10} - \frac{1}{60} \text{ (answer from 5a)} \\&= \frac{1}{4}\end{aligned}$$