GAMES

MATH CLUB

TAKING STUFF FROM A PILE

- For this kind of game question, usually the important point is to find certain things that do not change.
- Ex. Lisa and Max are playing a game. There are 9 rocks, they can take out one or two rocks at a time. The person that takes the last rock wins. Max is going first. Who has a winning strategy?

SLIGHTLY HARDER

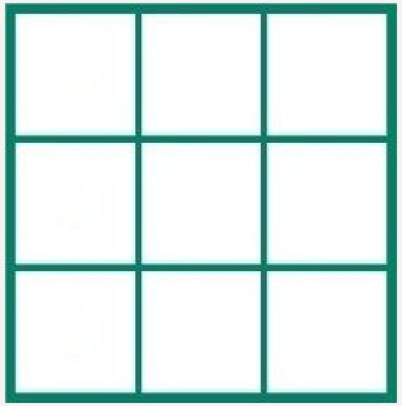
- Ex. Lisa and Max are playing a game. There are 8 rocks, they can take out one or two rocks at a time. The person that takes the last rock loses. Max is going first. Who has a winning strategy?
- Ex. Lisa and Max are playing a game. There are 2016 rocks, they can take out one, two, three or four rocks at a time. The person that takes the last rock loses. Max is going first. Who has a winning strategy?

A LITTLE TWIST

- Max and Lisa are putting coins on a round table and the coins can't overlap. Whoever can't put a coin on the desk loses. Max goes first, who had winning strategy?

CHESS BASED GAMES

- There is a 3 × 3 checkerboard, and nine cards with numbers 1,3,4,5,6,7,8,9,10 on them. Max and Lisa take turns to put a card onto the grid. Max calculates the sum of numbers in two columns on the left and the right, Lisa calculate sum of numbers of the top and bottom rows. The person with the larger sum wins. Max goes first. Who is guaranteed to win?
- What if Max uses one move, then Lisa has two, then Max has another one?



THE M × N CHECKERBOARD (M, N IS A NATURAL NUMBER)

Max and Lisa play a game, they each take a pen,. Max first draws a dot at the center of grid, then Lisa draws a dot at the center of an adjacent grid. Max then draws a dot on the centre of an adjacent grid of the grid Lisa drew a dot on, and they repeat the process. Whoever can't draw anymore loses. Who can win? (Note: already painted boxes are not allowed to be re-drawn)

2014 EUCLID LAST QUESTION

Fiona plays a game with jelly beans on the number line. Initially, she has N jelly beans, all at position 0. On each turn, she must choose one of the following moves:

• Type 1: She removes two jelly beans from position 0, eats one, and puts the other at position 1.

• Type i, where i is an integer with $i \geq 2$: She removes one jelly bean from position $i - 2$ and one jelly bean from position $i - 1$, eats one, and puts the other at position i.

The positions of the jelly beans when no more moves are possible is called the final state. Once a final state is reached, Fiona is said to have won the game if there are at most three jelly beans remaining, each at a distinct position and no two at consecutive integer positions. For example, if N = 7, Fiona wins the game with the sequence of moves

Type 1, Type 1, Type 2, Type 1, Type 3

which leaves jelly beans at positions 1 and 3. A different sequence of moves starting with N = 7 might not win the game.

- (a) Determine an integer N for which it is possible to win the game with one jelly bean left at position 5 and no jelly beans left at any other position.
- (b) Suppose that Fiona starts the game with a fixed unknown positive integer N. Prove that if Fiona can win the game, then there is only one possible final state.
- (c) Determine, with justification, the closest positive integer N to 2014 for which Fiona can win the game.