Probability

1. A 75 year old person has a 50% chance of living at least another 10 years. A 75 year old person has a 20% chance of living at least another 15 years. An 80 year old person has a 25% chance of living at least another 10 years. What is the probability that an 80 year old person will live at least another 5 years?

2. Billy and Crystal each have a bag of 9 balls. The balls in each bag are numbered from 1 to 9. Billy and Crystal each remove one ball from their own bag. Let b be the sum of the numbers on the balls remaining in Billy's bag. Let c be the sum of the numbers on the balls remaining in Crystal's bag. Determine the probability that b and c differ by a multiple of 4.

3. In the 4 × 4 grid shown, three coins are randomly placed in different squares. Determine the probability that no two coins lie in the same row or column.

4. Oi-Lam tosses three fair coins and removes all of the coins that come up heads. George then tosses the coins that remain, if any. Determine the probability that George tosses exactly one head.

5. A farmer has a flock of n sheep, where $2000 \leqslant n \leqslant 2100$. The farmer puts some number of the sheep into one barn and the rest of the sheep into a second barn. The farmer realizes that if she were to select two different sheep at random from her flock, the probability that they are in different barns is exactly 1/2. Determine the value of n.

6. Let k be a positive integer with k >=2. Two bags each contain k balls, labelled with the positive integers from 1 to k. Andree removes one ball from each bag. (In each bag, each ball is equally likely to be chosen.) Define P(k) to be the probability that the product of the numbers on the two balls that he chooses is divisible by k.

   a. Calculate P(10)
   b. Determine, with justification, a polynomial f(n) for which
      $P(n) >= f(n)/n^2$ for all positive integers n with n >= 2, and
      $P(n) =f(n)/n^2$ for infinitely many positive integers n with n >= 2.
      (A polynomial f(x) is an algebraic expression of the form $f(x) = a_m x^m + a_{m-1}x^{m+1} + ... + a_1x+ a_0$ for some integer m >= 0 and for some real numbers am; a_{m-1}..... a_1; a_0.)
   c. Prove there exists a positive integer m for which P(m) > 2016/m