Tuesday, FEBRUARY 12, 2002

Contest A

The MATHEMATICAL ASSOCIATION OF AMERICA
American Mathematics Competitions
Presented by the Akamai Foundation

AMC 12

53rd Annual American Mathematics Contest 12

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question, multiple choice test. Each question is followed by answers marked A,B,C,D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have **75 MINUTES** working time to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 20th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 26, 2002 or Tuesday, April 9, 2002. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. Compute the sum of all the roots of $(2x+3)(x-4)+(2x+3)(x-6)=0$.

(A) 7/2 **(B)** 4 **(C)** 5 **(D)** 7 **(E)** 13

2. Cindy was asked by her teacher to subtract 3 from a certain number and then divide the result by 9. Instead, she subtracted 9 and then divided the result by 3, giving an answer of 43. What would her answer have been had she worked the problem correctly?

(A) 15 **(B)** 34 **(C)** 43 **(D)** 51 **(E)** 138

3. According to the standard convention for exponentiation,

$$2^{2^{2^{2}}} = 2^{\left(2^{\left(2^{2}\right)}\right)} = 2^{16} = 65,536.$$

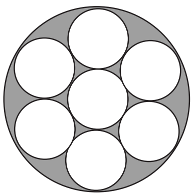
If the order in which the exponentiations are performed is changed, how many other values are possible?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

4. Find the degree measure of an angle whose complement is 25% of its supplement.

(A) 48 **(B)** 60 **(C)** 75 **(D)** 120 **(E)** 150

5. Each of the small circles in the figure has radius one. The innermost circle is tangent to the six circles that surround it, and each of those circles is tangent to the large circle and to its small-circle neighbors. Find the area of the shaded region.



(A) π **(B)** 1.5π **(C)** 2π **(D)** 3π **(E)** 3.5π

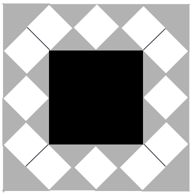
6. For how many positive integers m does there exist at least one positive integer n such that $m \cdot n \le m + n$?

(A) 4 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** infinitely many

7. If an arc of $45°$ on circle A has the same length as an arc of $30°$ on circle B, then the ratio of the area of circle A to the area of circle B is

(A) $\frac{4}{9}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{5}{6}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{9}{4}$

8. Betsy designed a flag using blue triangles (■), small white squares (□), and a red center square(■), as shown. Let B be the total area of the blue triangles, W the total area of the white squares, and R the area of the red square. Which of the following is correct?



(A) $B = W$ **(B)** $W = R$ **(C)** $B = R$ **(D)** $3B = 2R$ **(E)** $2R = W$

9. Jamal wants to store 30 computer files on floppy disks, each of which has a capacity of 1.44 megabytes (mb). Three of his files require 0.8 mb of memory each, 12 more require 0.7 mb each, and the remaining 15 require 0.4 mb each. No file can be split between floppy disks. What is the minimal number of floppy disks that will hold all the files?

(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

10. Sarah pours four ounces of coffee into an eight-ounce cup and four ounces of cream into a second cup of the same size. She then transfers half the coffee from the first cup to the second and, after stirring thoroughly, transfers half the liquid in the second cup back to the first. What fraction of the liquid in the first cup is now cream?

(A) 1/4 **(B)** 1/3 **(C)** 3/8 **(D)** 2/5 **(E)** 1/2

11. Mr. Earl E. Bird leaves his house for work at exactly 8:00 A.M. every morning. When he averages 40 miles per hour, he arrives at his workplace three minutes late. When he averages 60 miles per hour, he arrives three minutes early. At what average speed, in miles per hour, should Mr. Bird drive to arrive at his workplace precisely on time?

(A) 45 **(B)** 48 **(C)** 50 **(D)** 55 **(E)** 58

12. Both roots of the quadratic equation $x^2 - 63x + k = 0$ are prime numbers. The number of possible values of k is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** more than four

13. Two different positive numbers a and b each differ from their reciprocals by 1. What is $a+b$?

(A) 1 **(B)** 2 **(C)** $\sqrt{5}$ **(D)** $\sqrt{6}$ **(E)** 3

14. For all positive integers n, let $f(n) = \log_{2002} n^2$. Let

$$N = f(11) + f(13) + f(14).$$

Which of the following relations is true?

(A) $N > 1$ **(B)** $N = 1$ **(C)** $1 < N < 2$ **(D)** $N = 2$ **(E)** $N > 2$

15. The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. The largest integer that can be an element of this collection is

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

16. Tina randomly selects two distinct numbers from the set $\{1, 2, 3, 4, 5\}$, and Sergio randomly selects a number from the set $\{1, 2, \ldots, 10\}$. The probability that Sergio's number is larger than the sum of the two numbers chosen by Tina is

(A) 2/5 **(B)** 9/20 **(C)** 1/2 **(D)** 11/20 **(E)** 24/25

17. Several sets of prime numbers, such as $\{7, 83, 421, 659\}$, use each of the nine nonzero digits exactly once. What is the smallest possible sum such a set of primes could have?

(A) 193 **(B)** 207 **(C)** 225 **(D)** 252 **(E)** 477

18. Let C_1 and C_2 be circles defined by

$$(x-10)^2 + y^2 = 36$$

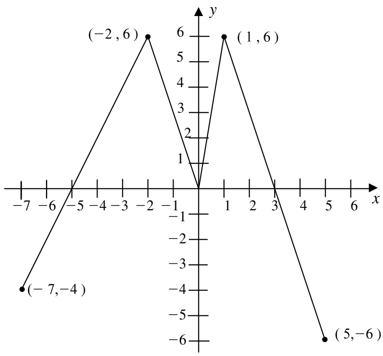
and

$$(x+15)^2 + y^2 = 81,$$

respectively. What is the length of the shortest line segment $\overline{PQ}$ that is tangent to C_1 at P and to C_2 at Q?

(A) 15 **(B)** 18 **(C)** 20 **(D)** 21 **(E)** 24

19. The graph of the function f is shown below. How many solutions does the equation $f(f(x)) = 6$ have?



(A) 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

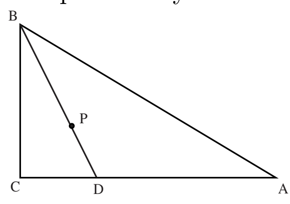
20. Suppose that a and b are digits, not both nine and not both zero, and the repeating decimal $0.\overline{ab}$ is expressed as a fraction in lowest terms. How many different denominators are possible?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 8 **(E)** 9

21. Consider the sequence of numbers: 4, 7, 1, 8, 9, 7, 6, ... For $n > 2$, the nth term of the sequence is the units digit of the sum of the two previous terms. Let S_n denote the sum of the first n terms of this sequence. The smallest value of n for which $S_n > 10,000$ is:

(A) 1992 **(B)** 1999 **(C)** 2001 **(D)** 2002 **(E)** 2004

22. Triangle ABC is a right triangle with $\angle ACB$ as its right angle, $m\angle ABC = 60°$, and $AB = 10$. Let P be randomly chosen inside $\triangle ABC$, and extend $\overline{BP}$ to meet $\overline{AC}$ at D. What is the probability that $BD > 5\sqrt{2}$?



(A) $\dfrac{2-\sqrt{2}}{2}$ **(B)** $\dfrac{1}{3}$ **(C)** $\dfrac{3-\sqrt{3}}{3}$ **(D)** $\dfrac{1}{2}$ **(E)** $\dfrac{5-\sqrt{5}}{5}$

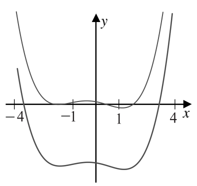
23. In triangle ABC, side $\overline{AC}$ and the perpendicular bisector of $\overline{BC}$ meet in point D, and $\overline{BD}$ bisects $\angle ABC$. If $AD = 9$ and $DC = 7$, what is the area of triangle ABD?

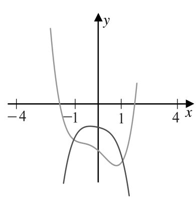
(A) 14 **(B)** 21 **(C)** 28 **(D)** $14\sqrt{5}$ **(E)** $28\sqrt{5}$

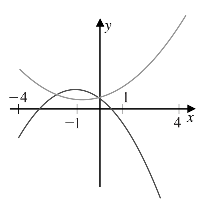
24. Find the number of ordered pairs of real numbers (a, b) such that $(a + bi)^{2002} = a - bi$.

(A) 1001 **(B)** 1002 **(C)** 2001 **(D)** 2002 **(E)** 2004

25. The nonzero coefficients of a polynomial P with real coefficients are all replaced by their mean to form a polynomial Q. Which of the following could be a graph of $y = P(x)$ and $y = Q(x)$ over the interval $-4 \le x \le 4$?

(A)


(B)


(C)


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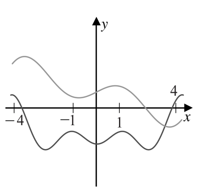
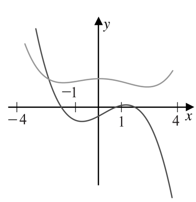


(E)


WRITE TO US!

Correspondence about the problems and solutions for this ***AMC 12*** *should be addressed to:*

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Lincoln, NE 68501-1606
Phone: 402-472-2257; Fax: 402-472-6087; email: titu@amc.unl.edu;

2002 AIME

The AIME will be held on Tuesday, March 26, 2002 with the alternate on April 9,2002. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score in the top 1% of the AMC 10 or receive a score of 100 or above on the AMC 12. Alternately, you must be in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) on Thursday through Sunday, May 9-12, 2002 in Cambridge, MA. The best way to prepare for the AIME and USAMO is to study previous years of these exams. Copies may be ordered as indicated below.

PUBLICATIONS

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Examinations: Each price is for one copy of an exam and its solutions for one year. Specify the years you want and how many copies of each. All prices effective to September 1, 2003.

- AMC 10 2000-2003/AHSME (AMC 12) 1989-2003, $1 per copy per year.
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2002

AMC 12 - Contest A

DO NOT OPEN UNTIL TUESDAY, FEBRUARY 12, 2002

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE FEBRUARY 12.** Nothing is needed from inside this package until February 12.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form A found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC Director, Titu Andreescu, no later than 24 hours following the examination.
4. Please Note: All Problems and Solutions are copyrighted; it is illegal to make copies or transmit them on the internet or world wide web without permission.
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