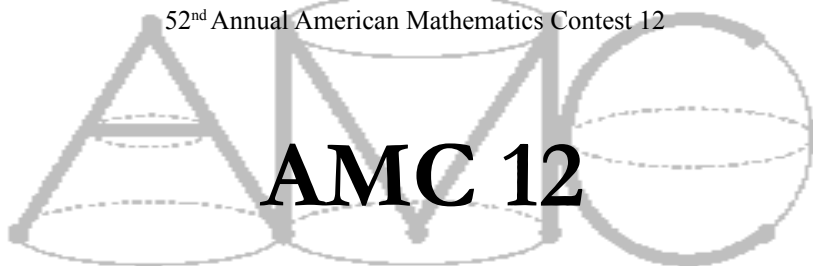


AMERICAN MATHEMATICS COMPETITIONS

52nd Annual American Mathematics Contest 12



AMC 12

TUESDAY, FEBRUARY 13, 2001

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question, multiple choice test. Each question is followed by answers marked A,B,C,D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. **SCORING:** You will receive 6 points for each correct answer, 2 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have **75 MINUTES** working time to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

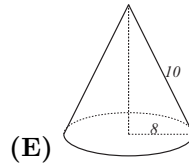
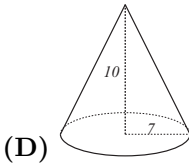
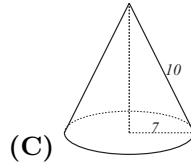
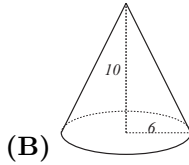
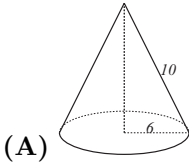
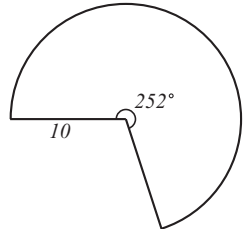
Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 19th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 27, 2001 or Tuesday, April 10, 2001. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

The publication, reproduction, or communication of the problems or solutions of the AMC 12 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Duplication at any time via copier, telephone, email, World Wide Web or media of any type is a violation of the

- The sum of two numbers is S . Suppose 3 is added to each number and then each of the resulting numbers is doubled. What is the sum of the final two numbers?
(A) $2S + 3$ (B) $3S + 2$ (C) $3S + 6$ (D) $2S + 6$ (E) $2S + 12$
- Let $P(n)$ and $S(n)$ denote the product and the sum, respectively, of the digits of the integer n . For example, $P(23) = 6$ and $S(23) = 5$. Suppose N is a two-digit number such that $N = P(N) + S(N)$. What is the units digit of N ?
(A) 2 (B) 3 (C) 6 (D) 8 (E) 9
- The state income tax where Kristin lives is levied at the rate of $p\%$ of the first \$28000 of annual income plus $(p + 2)\%$ of any amount above \$28000. Kristin noticed that the state income tax she paid amounted to $(p + 0.25)\%$ of her annual income. What was her annual income?
(A) \$28000 (B) \$32000 (C) \$35000 (D) \$42000 (E) \$56000
- The mean of three numbers is 10 more than the least of the numbers and less than the greatest. The median of the three numbers is 5. What is their sum?
(A) 5 (B) 20 (C) 25 (D) 30 (E) 36
- What is the product of all positive odd integers less than 10,000?
(A) $\frac{10000!}{(5000!)^2}$ (B) $\frac{10000!}{2^{5000}}$ (C) $\frac{9999!}{2^{5000}}$ (D) $\frac{10000!}{2^{5000} \cdot 5000!}$ (E) $\frac{5000!}{2^{5000}}$
- A telephone number has the form $ABC - DEF - GHIJ$, where each letter represents a different digit. The digits in each part of the number are in decreasing order; that is, $A > B > C$, $D > E > F$, and $G > H > I > J$. Furthermore, D , E , and F are consecutive even digits; G , H , I , and J are consecutive odd digits; and $A + B + C = 9$. Find A .
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8
- A charity sells 140 benefit tickets for a total of \$2001. Some tickets sell for full price (a whole dollar amount), and the rest sell for half price. How much money is raised by the full-price tickets?
(A) \$782 (B) \$986 (C) \$1158 (D) \$1219 (E) \$1449

8. Which of the cones below can be formed from a 252° sector of a circle of radius 10 by aligning the two straight sides?

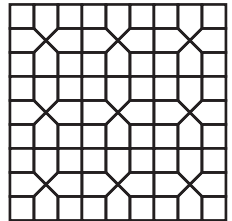


9. Let f be a function satisfying $f(xy) = f(x)/y$ for all positive real numbers x and y . If $f(500) = 3$, what is the value of $f(600)$?

- (A) 1 (B) 2 (C) $\frac{5}{2}$ (D) 3 (E) $\frac{18}{5}$

10. The plane is tiled by congruent squares and congruent pentagons as indicated. The percent of the plane that is enclosed by the pentagons is closest to

- (A) 50 (B) 52 (C) 54 (D) 56 (E) 58



11. A box contains exactly five chips, three red and two white. Chips are randomly removed one at a time without replacement until all the red chips are drawn or all the white chips are drawn. What is the probability that the last chip drawn is white?

- (A) $\frac{3}{10}$ (B) $\frac{2}{5}$ (C) $\frac{1}{2}$ (D) $\frac{3}{5}$ (E) $\frac{7}{10}$

12. How many positive integers not exceeding 2001 are multiples of 3 or 4 but not 5?

- (A) 768 (B) 801 (C) 934 (D) 1067 (E) 1167

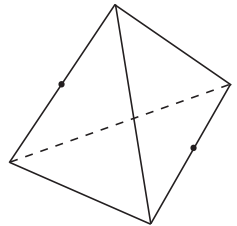
13. The parabola with equation $y = ax^2 + bx + c$ and vertex (h, k) is reflected about the line $y = k$. This results in the parabola with equation $y = dx^2 + ex + f$. Which of the following equals $a + b + c + d + e + f$?

- (A) $2b$ (B) $2c$ (C) $2a + 2b$ (D) $2h$ (E) $2k$

14. Given the nine-sided regular polygon $A_1A_2A_3A_4A_5A_6A_7A_8A_9$, how many distinct equilateral triangles in the plane of the polygon have at least two vertices in the set $\{A_1, A_2, \dots, A_9\}$?

- (A) 30 (B) 36 (C) 63 (D) 66 (E) 72

15. An insect lives on the surface of a regular tetrahedron with edges of length 1. It wishes to travel on the surface of the tetrahedron from the midpoint of one edge to the midpoint of the opposite edge. What is the length of the shortest such trip? (Note: Two edges of a tetrahedron are *opposite* if they have no common endpoint.)



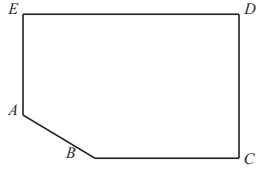
- (A) $\frac{1}{2}\sqrt{3}$ (B) 1 (C) $\sqrt{2}$ (D) $\frac{3}{2}$ (E) 2

16. A spider has one sock and one shoe for each of its eight legs. In how many different orders can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

- (A) $8!$ (B) $2^8 8!$ (C) $(8!)^2$ (D) $\frac{16!}{2^8}$ (E) $16!$

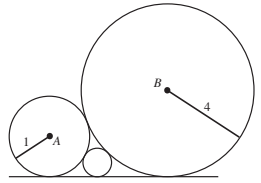
17. A point P is selected at random from the interior of the pentagon with vertices $A = (0, 2)$, $B = (4, 0)$, $C = (2\pi + 1, 0)$, $D = (2\pi + 1, 4)$, and $E = (0, 4)$. What is the probability that $\angle APB$ is obtuse?

- (A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{5}{16}$ (D) $\frac{3}{8}$ (E) $\frac{1}{2}$



18. A circle centered at A with a radius of 1 and a circle centered at B with a radius of 4 are externally tangent. A third circle is tangent to the first two and to one of their common external tangents as shown. The radius of the third circle is

- (A) $\frac{1}{3}$ (B) $\frac{2}{5}$ (C) $\frac{5}{12}$ (D) $\frac{4}{9}$ (E) $\frac{1}{2}$



19. The polynomial $P(x) = x^3 + ax^2 + bx + c$ has the property that the mean of its zeros, the product of its zeros, and the sum of its coefficients are all equal. If the y -intercept of the graph of $y = P(x)$ is 2, what is b ?

- (A) -11 (B) -10 (C) -9 (D) 1 (E) 5

20. Points $A = (3, 9)$, $B = (1, 1)$, $C = (5, 3)$, and $D = (a, b)$ lie in the first quadrant and are the vertices of quadrilateral $ABCD$. The quadrilateral formed by joining the midpoints of \overline{AB} , \overline{BC} , \overline{CD} , and \overline{DA} is a square. What is the sum of the coordinates of point D ?

- (A) 7 (B) 9 (C) 10 (D) 12 (E) 16

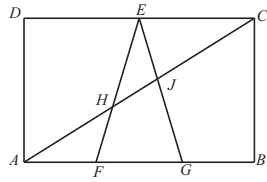
21. Four positive integers a , b , c , and d have a product of $8!$ and satisfy

$$\begin{aligned} ab + a + b &= 524, \\ bc + b + c &= 146, \text{ and} \\ cd + c + d &= 104. \end{aligned}$$

What is $a - d$?

- (A) 4 (B) 6 (C) 8 (D) 10 (E) 12

22. In rectangle $ABCD$, points F and G lie on \overline{AB} so that $AF = FG = GB$ and E is the midpoint of \overline{DC} . Also, \overline{AC} intersects \overline{EF} at H and \overline{EG} at J . The area of rectangle $ABCD$ is 70. Find the area of triangle EHJ .



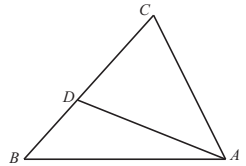
- (A) $\frac{5}{2}$ (B) $\frac{35}{12}$ (C) 3 (D) $\frac{7}{2}$ (E) $\frac{35}{8}$

23. A polynomial of degree four with leading coefficient 1 and integer coefficients has two real zeros, both of which are integers. Which of the following can also be a zero of the polynomial?

- (A) $\frac{1+i\sqrt{11}}{2}$ (B) $\frac{1+i}{2}$ (C) $\frac{1}{2} + i$ (D) $1 + \frac{i}{2}$ (E) $\frac{1+i\sqrt{13}}{2}$

24. In triangle ABC , $\angle ABC = 45^\circ$. Point D is on \overline{BC} so that $2 \cdot BD = CD$ and $\angle DAB = 15^\circ$. Find $\angle ACB$.

- (A) 54° (B) 60° (C) 72° (D) 75° (E) 90°



25. Consider sequences of positive real numbers of the form $x, 2000, y, \dots$, in which every term after the first is 1 less than the product of its two immediate neighbors. For how many different values of x does the term 2001 appear somewhere in the sequence?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) more than 4

WRITE TO US!

Correspondence about the problems and solutions for this AMC 12 should be addressed to:

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Phone: 409-845-2831; Fax: 409-845-6028 eMail: vince@math.tamu.edu;

Orders for any of the publications listed below should be addressed to:

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University of Nebraska, P.O. Box 81606
Lincoln, NE 68501-1606
Phone: 402-472-2257; Fax: 402-472-6087; email: titu@amc.unl.edu; www.unl.edu/amc

2001 AIME

The AIME will be held on Tuesday, March 27, 2001 with the alternate on April 10, 2001. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate if you receive a score of 100 or above on the AMC 12, alternately, you must be in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) on Tuesday, May 1, 2001. The best way to prepare for the AIME and USAMO is to study previous years of these exams. Copies may be ordered as indicated below.

PUBLICATIONS

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2001

AMC 12

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****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE FEBRUARY 13.** Nothing is needed from inside this package until February 13.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form A found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC Director, Titu Andreescu, no later than 24 hours following the examination.
4. Please Note: All Problems and Solutions are copyrighted; it is illegal to make copies or transmit them on the internet or world wide web without permission.
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